Rotational Kinematic Formulas

What do the Rotational Kinematic formulas mean?

The Rotational Kinematic formulas are the same 4 formulas we had for linear variables ($\Delta x, v_i, v_f, a, t$) but replaced with their angular counterparts ($\Delta\theta, \omega_i, \omega_f, \alpha, t$).

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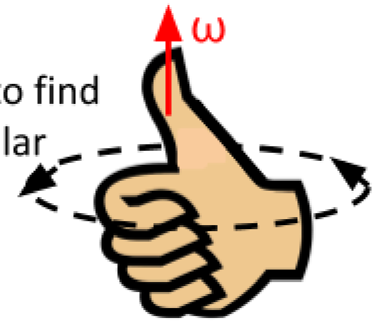
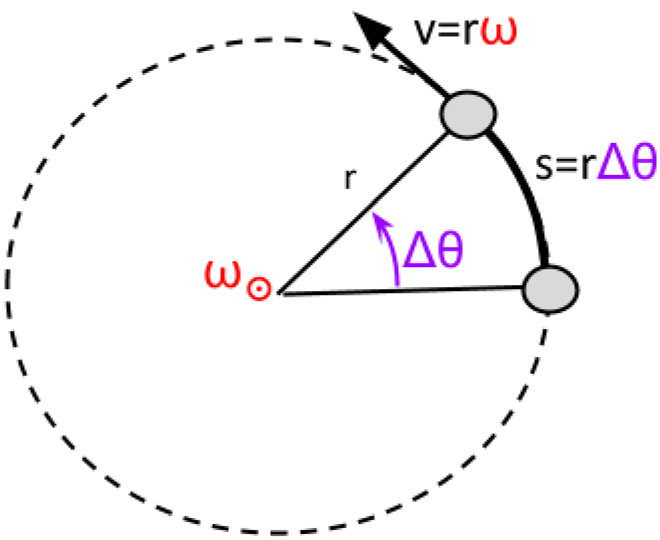
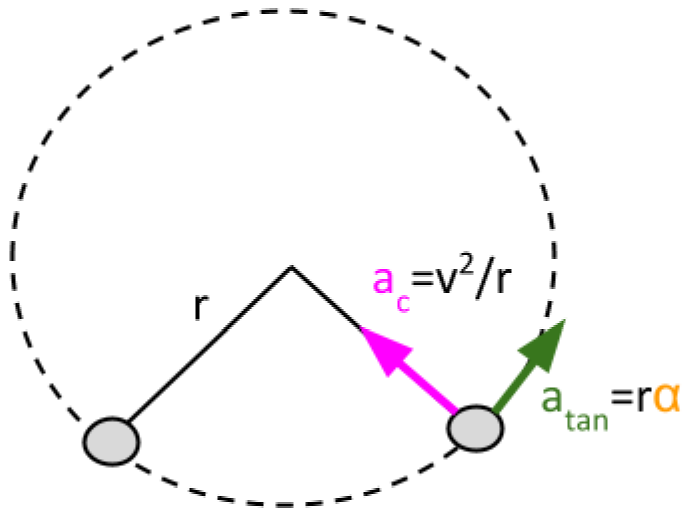
4 Rotational Kinematic Formulas

1. $\omega_f = \omega_i + \alpha t$
2. $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
3. $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
4. $(\omega_f + \omega_i)/2 = \Delta\theta / t$

Right hand rule to find direction of angular velocity ω.







Example Question:

Question: An object is rotating in a circle at a constant rate. Which best describes the accelerations of the object?

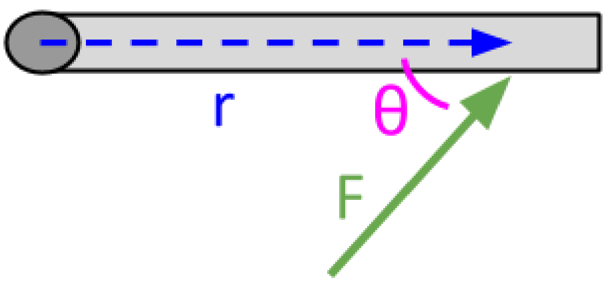
	Angular acc.	Tangential acc.	Centripetal acc.
A.	Non-zero	Non-zero	Zero
B.	Zero	Zero	Zero
C.	Non-zero	Non-zero	Non-zero
D.	Zero	Zero	Non-zero

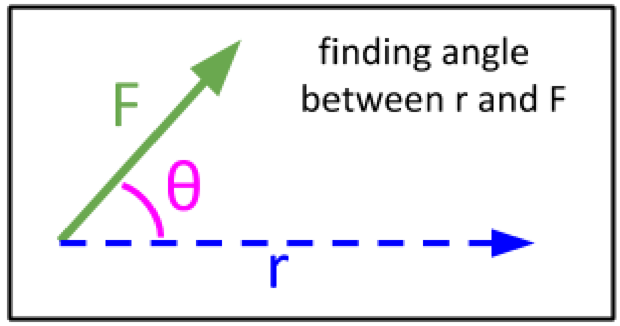
Torque τ Units: **Nm** Vector? **Yes**

What does torque mean?

Force is what causes acceleration. Torque is what causes angular acceleration.

$$\tau = rF\sin\theta$$





Conditions for equilibrium

"Translational" Equilibrium: $\Sigma F = 0$

"Rotational" Equilibrium: $\Sigma \tau = 0$

Example Question:

Question: A ball is rotating in a circle and slowing down. In reference to the direction of the ball's velocity v and angular velocity ω, what are the directions of the net torque, tangential force, and centripetal force on the ball?

	Net torque	Tangential force	Centripetal force
A.	Opposite to ω	no direction (zero)	perpendicular to v
B.	Same direction as ω	Opposite to v	no direction (zero)
C.	Opposite to ω	Opposite to v	perpendicular to v
D.	Perpendicular to ω	no direction (zero)	no direction (zero)

Rotational Inertia I

Units: kg m^2 **Vector?** No

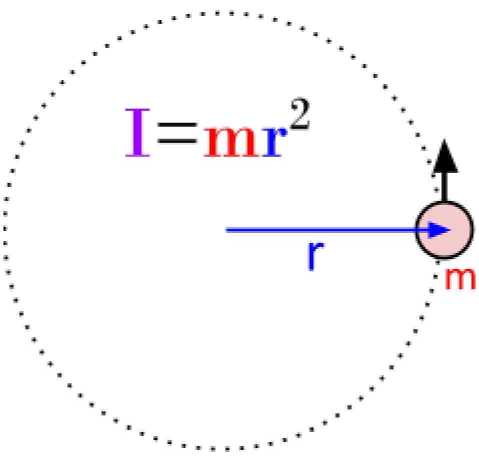
What does Rotational Inertia mean?

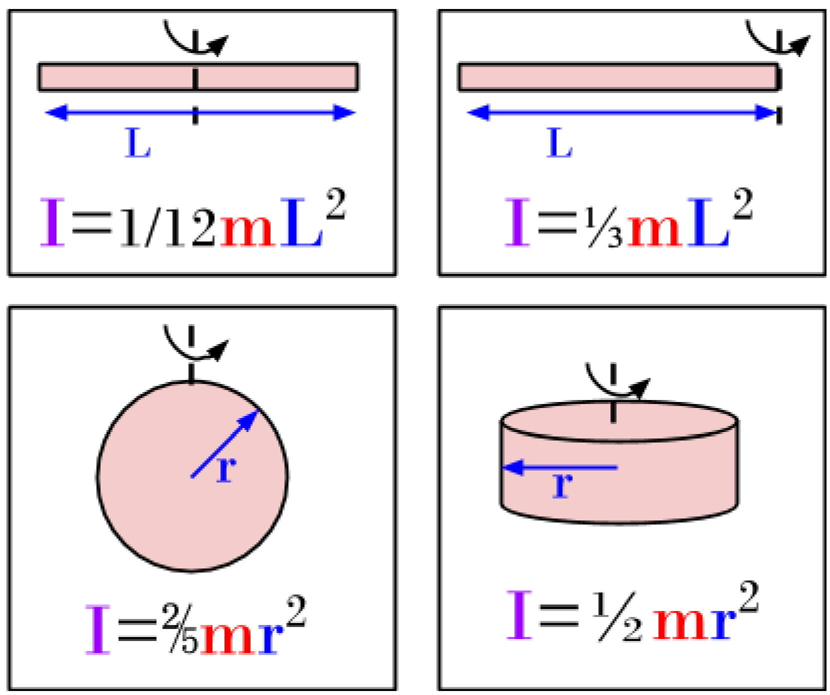
An object with a larger Rotational inertia will be harder to get rotating, and harder to stop rotating. Rotational inertia is also called "Moment of inertia".

An object will have a larger rotational inertia if its mass is distributed far from **the axis**.
An object will have a smaller rotational inertia if its mass is distributed close to **the axis**.

$I = mr^2$ (single mass going in a circle of a single radius)

$I = \Sigma mr^2$ (multiple individual masses going in circles of different radius)

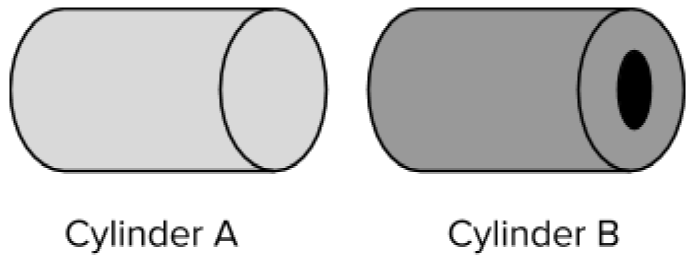




Example Question:

Question: Two cylinders are allowed to roll without slipping down a hill from rest. The mass of cylinder A is distributed evenly throughout the cylinder. Cylinder B is made from a more dense material and has a hollow center with the mass surrounding the central axis as seen in the diagram below. The masses and radii of each cylinder are the same. Which cylinder will reach the bottom of the hill first?

A. Cylinder A
B. Cylinder B
C. They tie
D. The densities are needed to say

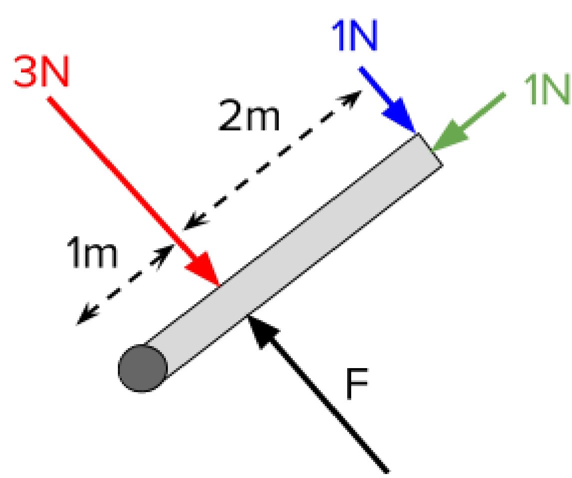


Answer: A

Example Question:

Question: Different forces are applied to a rod which can rotate about an axis at its end. How large would the force **F** have to be in order for the rod to be in rotational equilibrium?

A. 3N
B. 4N
C. 6N
D. 10N



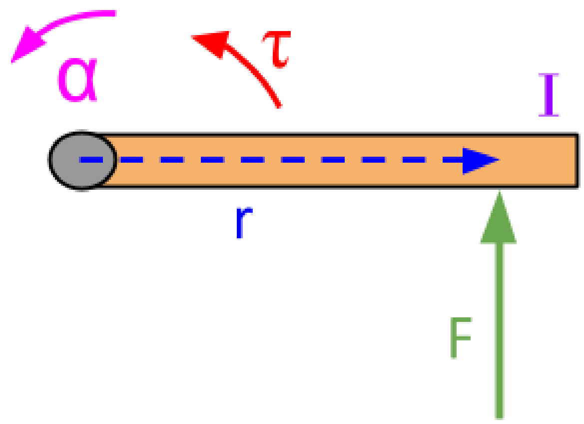
Answer:C

Angular version of Newton's Second Law

What does the Angular version of Newton's Second Law mean?

The angular version of Newton's Second Law says that the **angular acceleration** is proportional to the **net torque**, and inversely proportional to the **rotational inertia**.

$$\alpha = \Sigma\tau / I$$

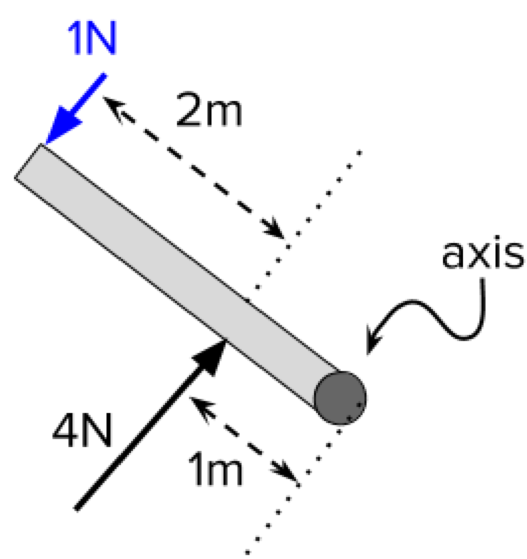


Warning: Torque is a vector, so it can be positive (CCW) or negative (CW).

Example Question:

Question: The rod shown below has a rotational inertia of 2 kg m^2 and the forces acting on it as shown. What is the magnitude of the angular acceleration of the rod?

A. 0.5 rad/s^2
B. 1.0 rad/s^2
C. 1.5 rad/s^2
D. 2.0 rad/s^2



Answer: A

Rotational Kinetic Energy

Units: kg m^2/s^2 **Vector?** No

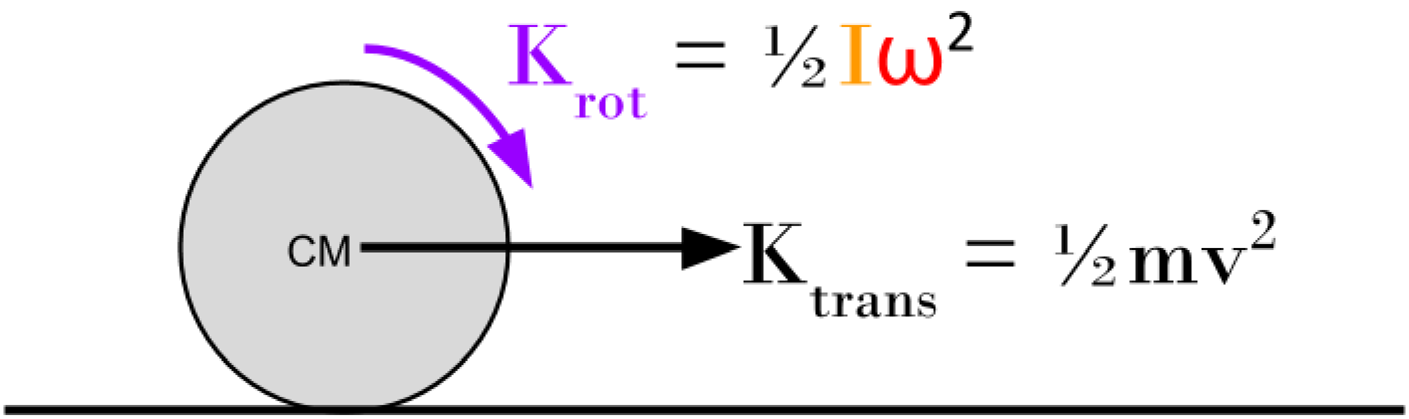
What does Rotational Kinetic Energy mean?

If an object is rotating it has rotational kinetic energy.

If the center of mass of the object is moving, and the object is rotating, it will have regular **translational kinetic energy** and rotational kinetic energy.

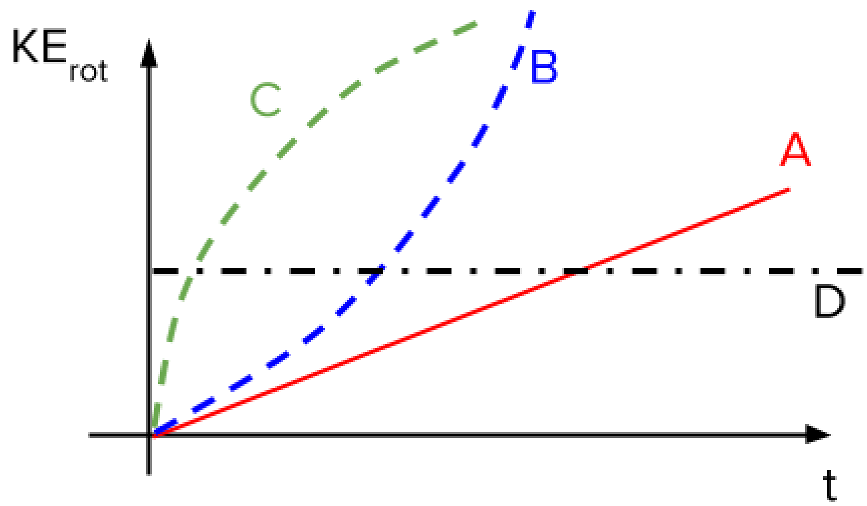
$K_{rotational} = ½I\omega^2$ (if the object is rotating with angular velocity ω)

$K_{translational} = ½mv^2$ (if the center of mass of the object is moving with speed v)



Example Question:

Question: A constant torque is exerted on a cylinder that is initially at rest which can rotate about an axis through its center. Which curve best gives the rotational kinetic energy of the cylinder as a function of time?



A. A
B. B
C. C
D. D

Answer: B

Angular Momentum

Units: **kg m²/s** — **Vector?** **Yes**

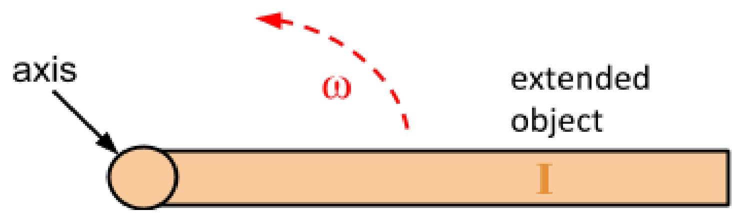
What does Angular Momentum mean?

Angular momentum is conserved if there is **no external torque**.

Even a point mass moving in a straight line can have angular momentum (since if it hits something it can cause that thing to start rotating.)

$L = I\omega$ (extended objects)

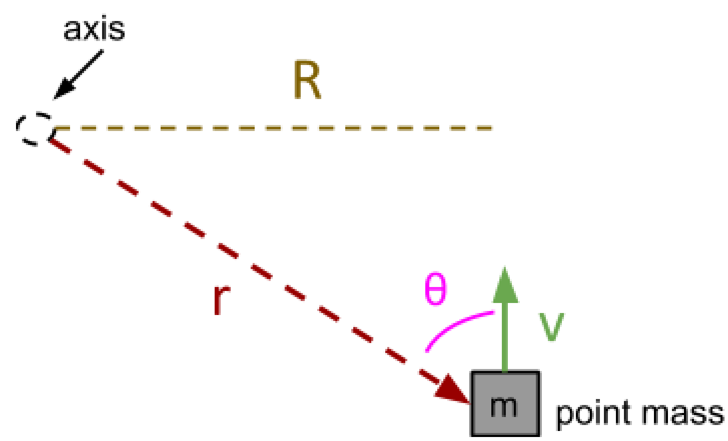
L = angular momentum
I = rotational inertia
ω = angular velocity



$L = mv(r\sin\theta)$ (point masses)

$L = mv(R)$

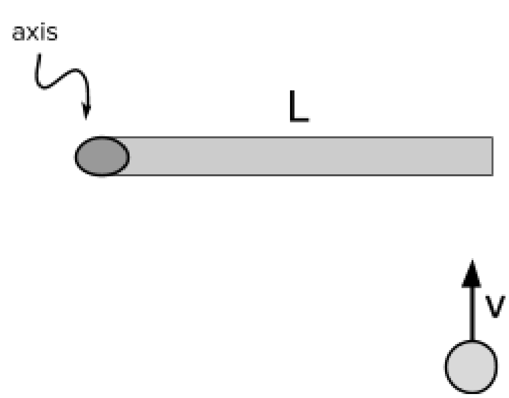
L = angular momentum
m = mass moving with speed v
v = velocity of the mass
r = distance from the axis to the mass m
θ = angle between r and the velocity of the mass
P = point of closest approach, which is equal to $r\sin\theta$



Example Question:

Question: A clay sphere of mass M is heading toward a rod of mass 3M and length L with a speed v. The rod is free to rotate about an axis at its end. If the clay slicks to the end of the rod, what is the angular velocity of the rod after the clay sticks to the rod? (The moment of inertia of a rod about its end is $\frac{1}{3}mL^2$)

A. $\frac{v}{2L}$
B. $\frac{2v}{L}$
C. $\frac{3v}{2L}$
D. $\frac{2v}{3L}$



Answer: A

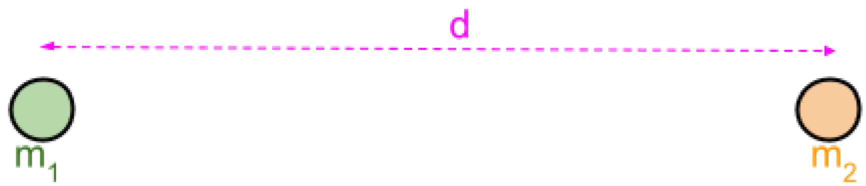
Gravitational Potential Energy U_g Units: J   Vector? No

What does Gravitational Potential Energy U_g mean?

For a region where the gravitational field is constant we can use $U_g = mgh$, but if the gravitational field g is varying we have to use the more general formula for gravitational potential energy U_g.

Any time two masses (i.e. a planet and a moon) are near each other, they will have a gravitational potential energy. But this energy will always be negative since it is defined to be zero when the masses are infinitely far away from each other.

$$U_g = -G\frac{m_1 m_2}{d}$$



U_g is the gravitational potential energy

$G = 6.67\times10^{-11}\ N\ m^2/kg^2$

m_1 is the mass of one of the masses

m_2 is the mass of the other mass

d is the distance from the mass m_1 to the mass m_2

Warning:

The potential energy will always be negative (or zero) because of how it is defined, but U_g can still convert into K since U_g can decrease by becoming more and more negative.

Example Question:

Question: Two spheres of radius R and mass M are falling toward each other due to gravitational attraction. If the surface to surface distance between the spheres starts out as 4R, and ends up as 2R, how much kinetic energy was gained by the system?

A. $G\frac{M^2}{R}$

B. $G\frac{M^2}{2R}$

C. $G\frac{M^2}{6R}$

D. $G\frac{M^2}{12R}$