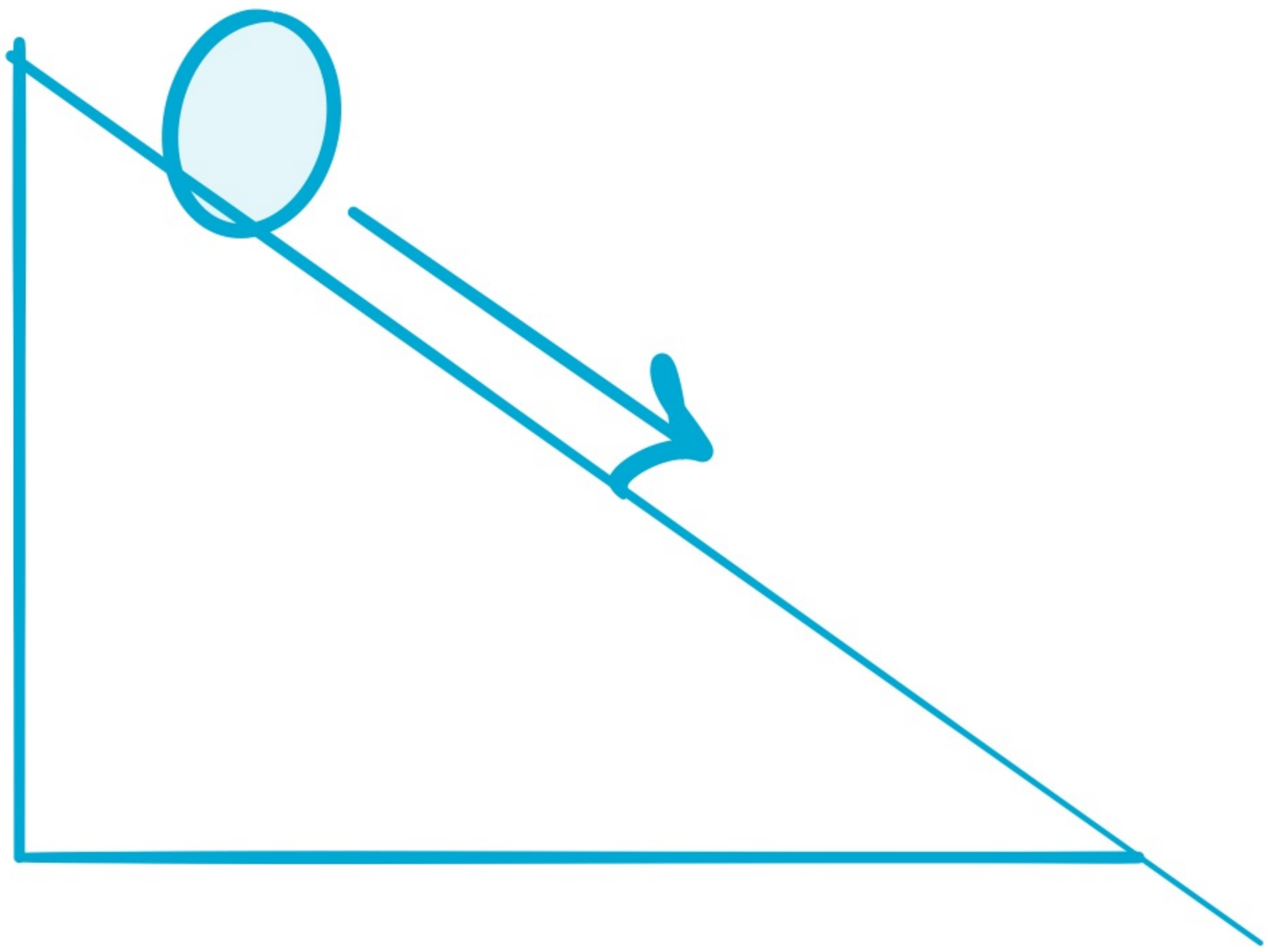
AP Physics:

Rotational Motion

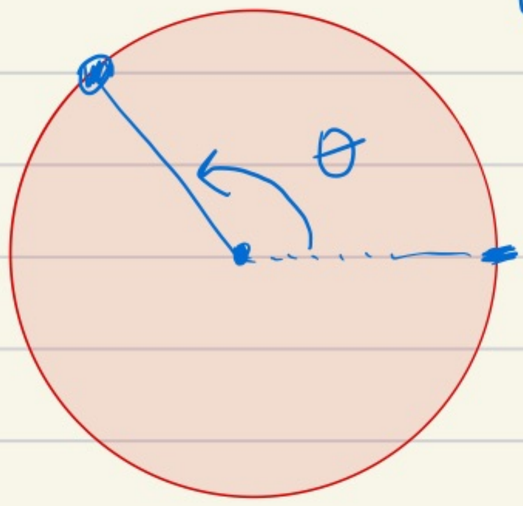


Rotational Motion

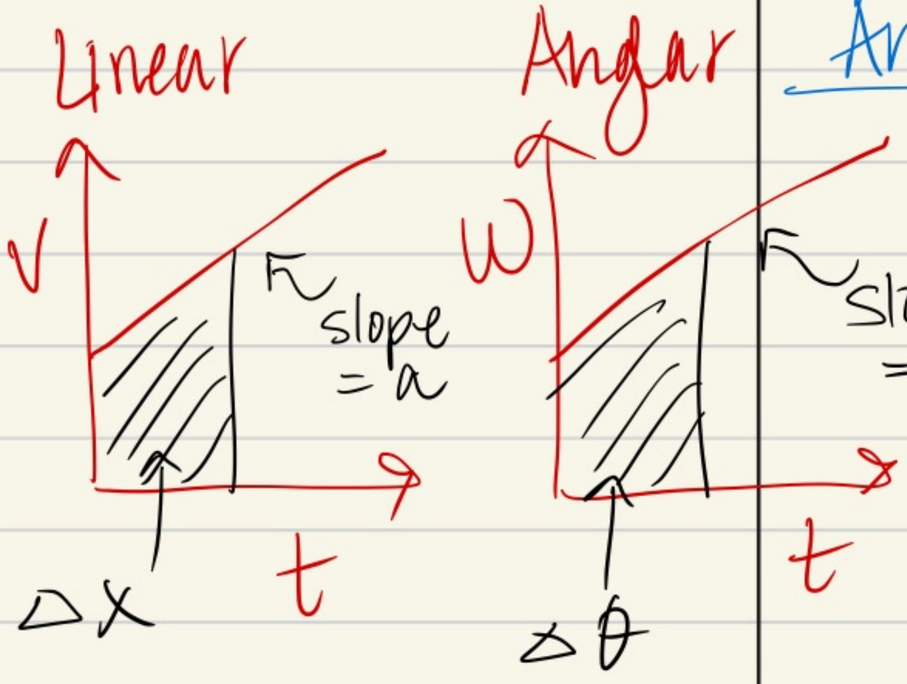
Ch. 8: Algebra
Ch. 10: Calculus

[Radians] $\frac{180}{\pi}$.

$\pi_{rad} = 180°$
$2\pi\ rad = 360°$



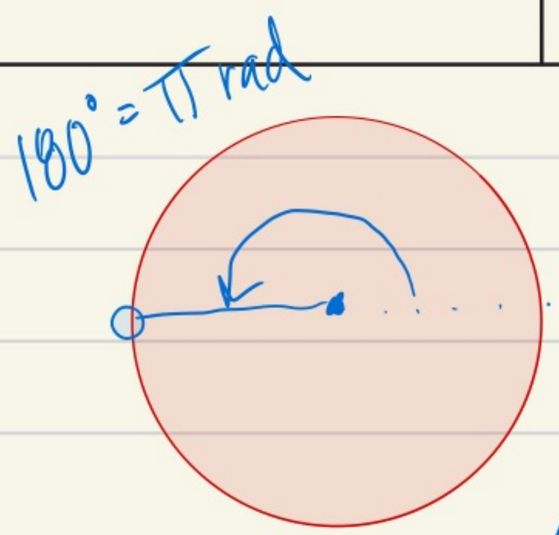
Linear (translational)	Rotational (angular)
$\Delta x = x_f - x_i$	$\Delta\theta = \theta_f - \theta_i$ (radians)
$\vec{v} = \frac{\Delta x}{\Delta t}$	$\vec{\omega} = \frac{\Delta\theta}{\Delta t}$
$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$	$\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}$



Angular Kinematics

#1 $\omega_f = \omega_i + \alpha\Delta t$
#2 $\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$
#3 $\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha\Delta t^2$
#4 $\Delta\theta = \omega_f \Delta t - \frac{1}{2}\alpha\Delta t^2$
#5 $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

↑ Assume const accel



$\omega_f = 1.57 \frac{rad}{s}$

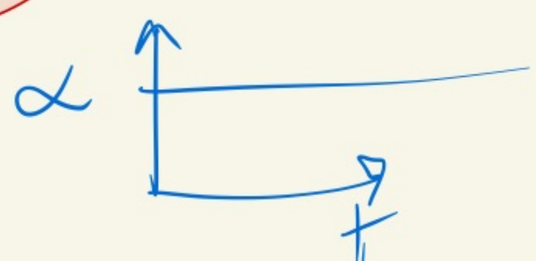
$\omega_0 = 0$
$\Delta t = 4s$

$\omega_{Ave} = ? = \frac{\Delta\theta}{\Delta t} = \frac{\pi\ rad}{4s}$

$\omega_{ave} = 0.785 \frac{rad}{s}$

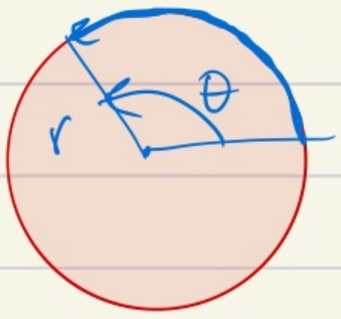
$\alpha = ?$
$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{1.57\ rad/s - 0}{4s}$

$0.393 \frac{rad}{s^2}$



ing & ave

How to translate between angular & linear



arc length = $s = r \cdot \Delta\theta$

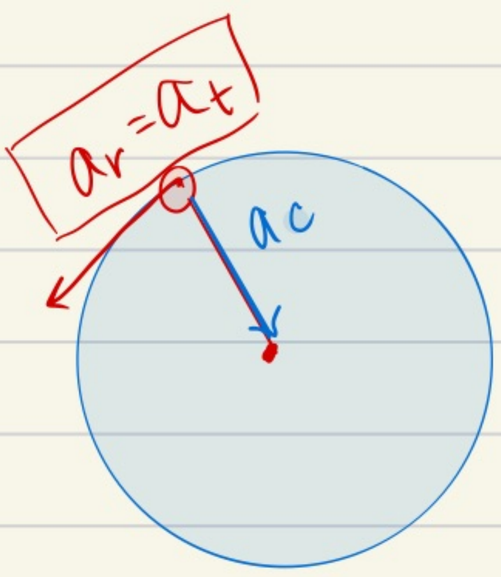
$$r \cdot w = r \cdot \frac{\Delta\theta}{\Delta t} = \frac{r \cdot \Delta\theta}{\Delta t} = \frac{s}{\Delta t}$$

Speed — NOT velocity

$v = rw$

linear speed, radius, angular vel.

t: tang
r: radial



$$r \cdot \alpha = \frac{\Delta w}{\Delta t} \cdot r = \frac{r(w_f - w_i)}{\Delta t} = \frac{v_f - v_i}{\Delta t} = a_{tan}$$

(speed)

$$a_{tan} = r\alpha$$

The 4m long bar below starts from rest and rotates through 5 revolutions with a constant angular acceleration of 30 rad/s^2.

a. How long did it take to make 5 revolutions?

b. What was the angular velocity after rotating 5 revolutions?

① $\omega_0 = 0$

5 rev $= 5 \cdot 2\pi$

② $\Delta\theta = 10\pi$ rad.

③ $\alpha = 30 \frac{rad}{s^2}$

$t = ?$

#3 $\Delta\theta = \omega_0 \Delta t + \frac{1}{2}\alpha \Delta t^2$ ($\omega_0 = 0$)

$10\pi \text{ rad} = \frac{1}{2}\left(30 \frac{rad}{s^2}\right) \cdot \Delta t^2$

$\Delta t^2 = \frac{(10\pi \text{ rad})(2)}{30 \text{ rad/s}^2}$

$\Delta t = \sqrt{\quad}$

$\Delta t = \frac{\sqrt{6}}{3} s = 1.45 s$

⑥ $\omega_f = ?$

$\omega_f = \omega_0 + \alpha \Delta t$

$\omega_f = 0 + \frac{30 rad}{s^2}(1.45 s)$

$\omega_f = 43.5 \frac{rad}{s}$

The 4m long bar below starts with an angular velocity of 40 rad/s and decelerates with a constant deceleration to a stop after rotating through 20 revolutions.

a. How fast is the edge of the bar moving initially in m/s?

b. What was the angular acceleration of the bar?

(a)

$\omega_i = 40 \text{ rad/s}$

$\omega_f = 0$

$\Delta\theta = 20 \cdot 2\pi \text{ rad}$
$= 40\pi \text{ rad}$

$v_i = r \cdot \omega_i =$
$(4m)(40 \text{ rad/s})$

$v_i = 160 \text{ m/s}$ or
200 m/s

(b). $\alpha = ?$ #5 $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

$0 = \omega_i^2 + 2\alpha\Delta\theta$

$0 = \left(\frac{40 \text{ rad}}{s}\right)^2 + 2(\alpha)(40\pi)$

$2 \cdot \alpha \cdot 40\pi = -(40)^2$

$\alpha = \frac{-(40)^2}{2 \cdot 40\pi} = -6.37 \frac{\text{rad}}{s^2}$ ☺

HW alg ch. 8 1, 4, 7, 15, 16

calc ch. 10 1, 4, 5, 16, 17