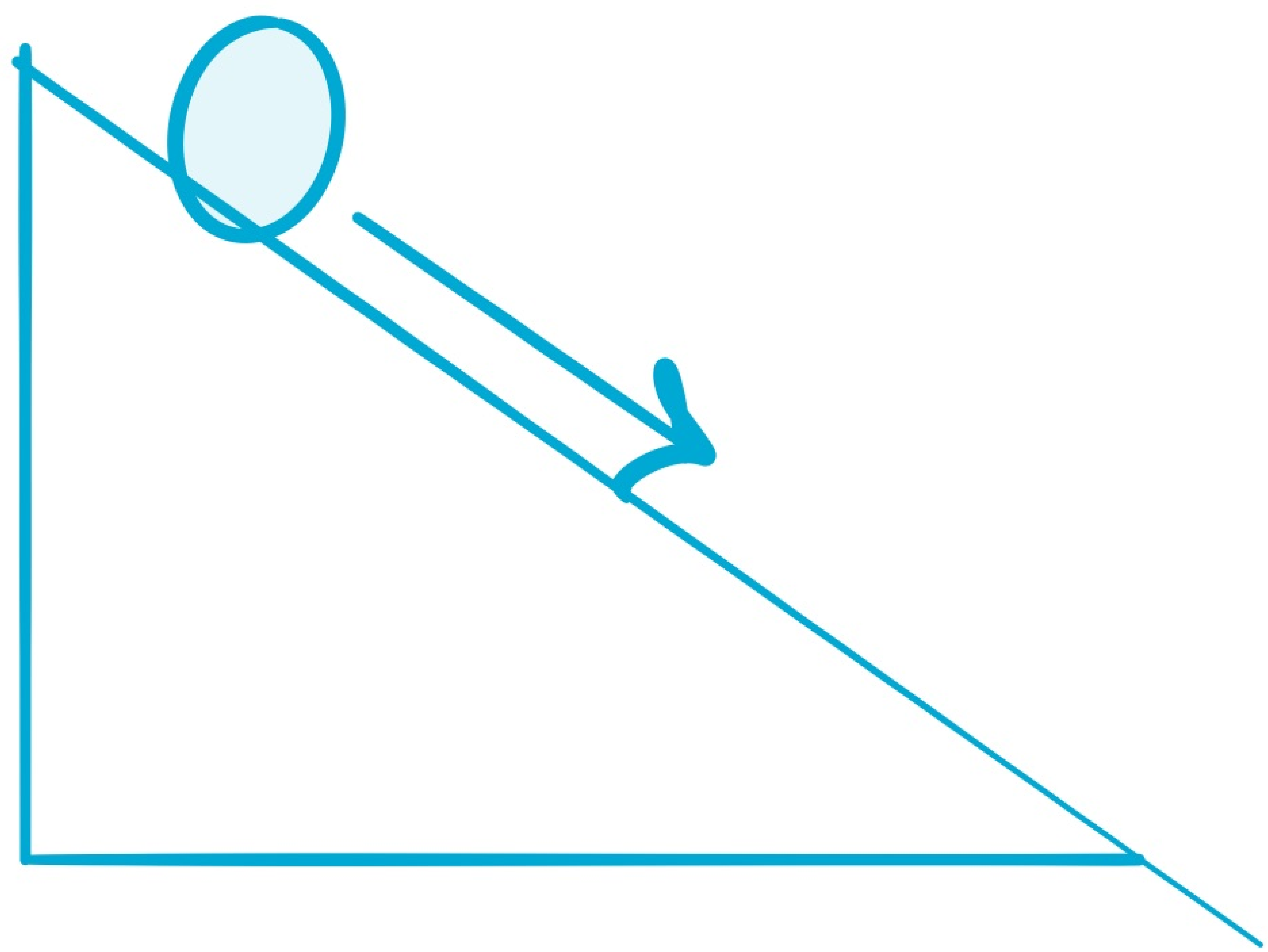
AP Physics:

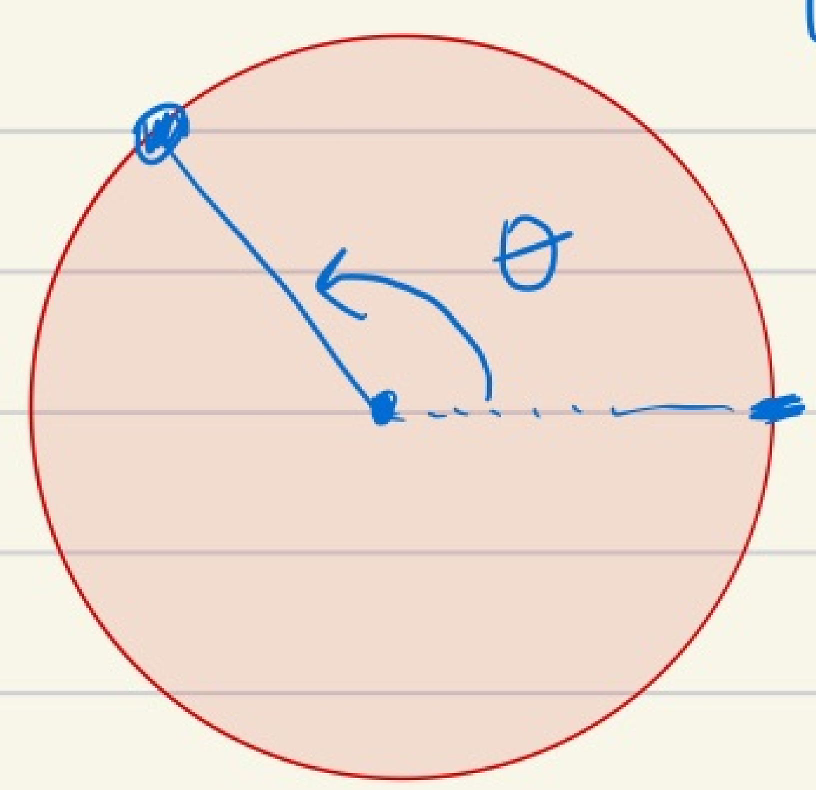
Rotational Motion



Ch. 8: Algebra
Ch. 10: Calculus

Rotational Motion

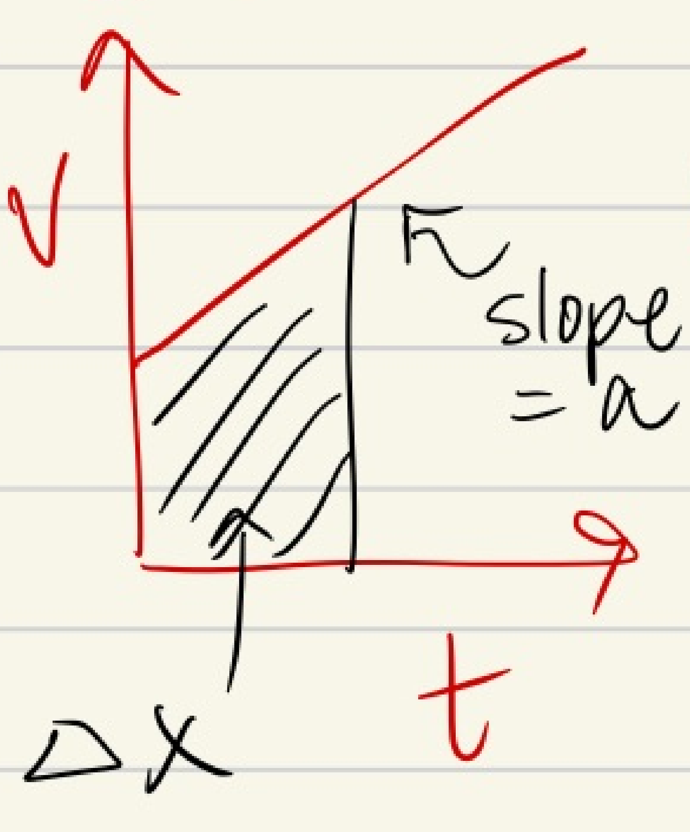
[Radians] $\frac{180}{\pi}$



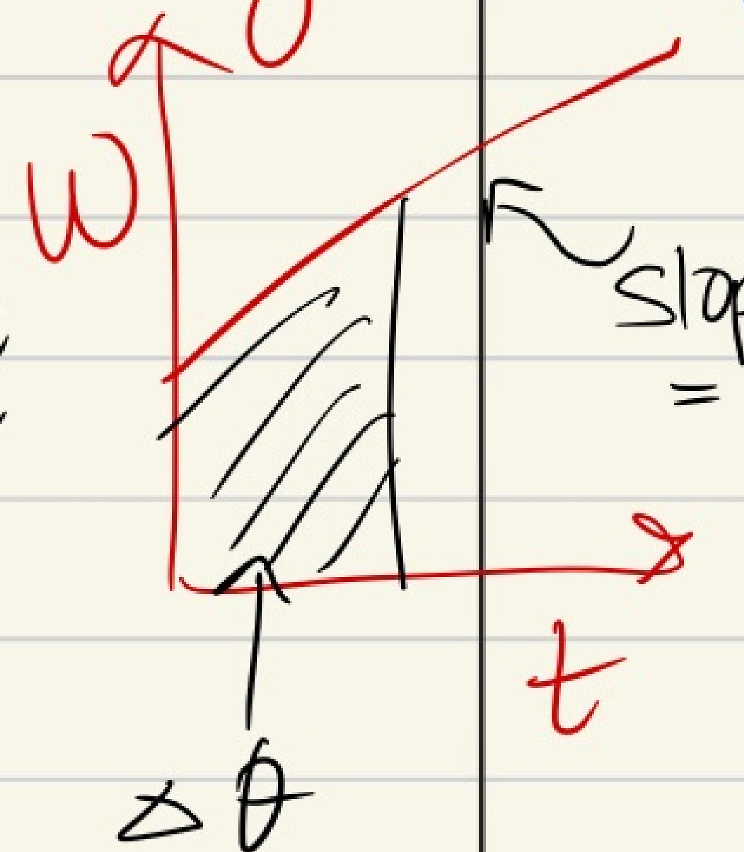
$\pi_{rad} = 180°$
$2\pi \text{ rad} = 360°$

Linear (translational)		Rotational (angular)
$\Delta x = x_f - x_i$		$\Delta\theta = \theta_f - \theta_i$ (radians)
$\vec{v} = \frac{\Delta x}{\Delta t}$	⟷	$\vec{\omega} = \frac{\Delta\theta}{\Delta t}$
$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$	⟷	$\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}$

Linear



Angular

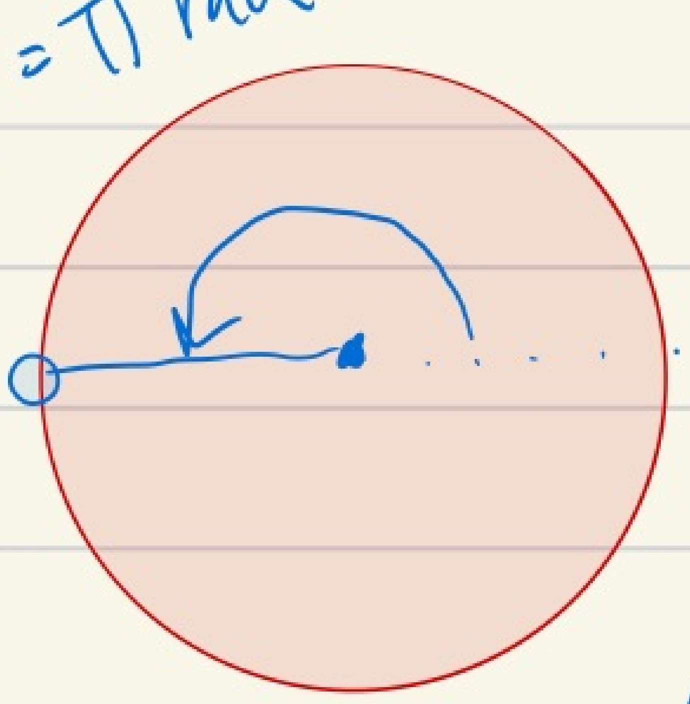


Angular Kinematics

#1 $\omega_f = \omega_i + \alpha\Delta t$
#2 $\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$
#3 $\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2$
#4 $\Delta\theta = \omega_f\Delta t - \frac{1}{2}\alpha\Delta t^2$
#5 $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Assume $\boxed{\text{const accel}}$

$180° = \pi \text{ rad}$



$\omega_f = 1.57 \frac{rad}{s}$

$\omega_0 = 0$
$\Delta t = 4s$

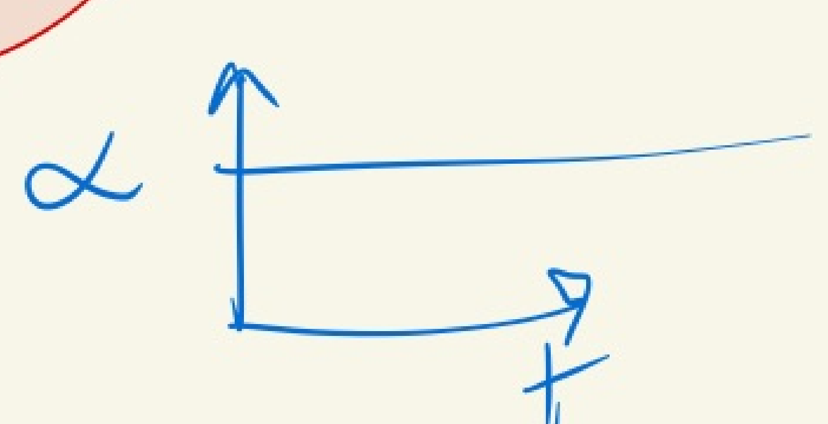
$\omega_{Ave} = ? = \frac{\Delta\theta}{\Delta t} = \frac{\pi \text{ rad}}{4s}$

$\omega_{ave} = 0.785 \frac{rad}{s}$

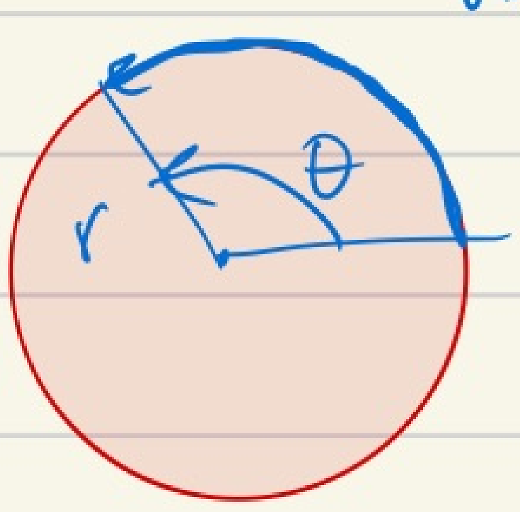
$\alpha = ?$

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{1.57\, rad/s - 0}{4s} = \boxed{0.393 \frac{rad}{s^2}}$

ins & ave



How to translate between angular & Linear



arclength $= \boxed{s = r \cdot \Delta\theta}$

$$r \cdot \omega = \frac{\Delta\theta}{\Delta t} \cdot r = r \cdot \frac{\Delta\theta}{\Delta t} = \boxed{\frac{s}{\Delta t}}$$

speed

NOT velocity

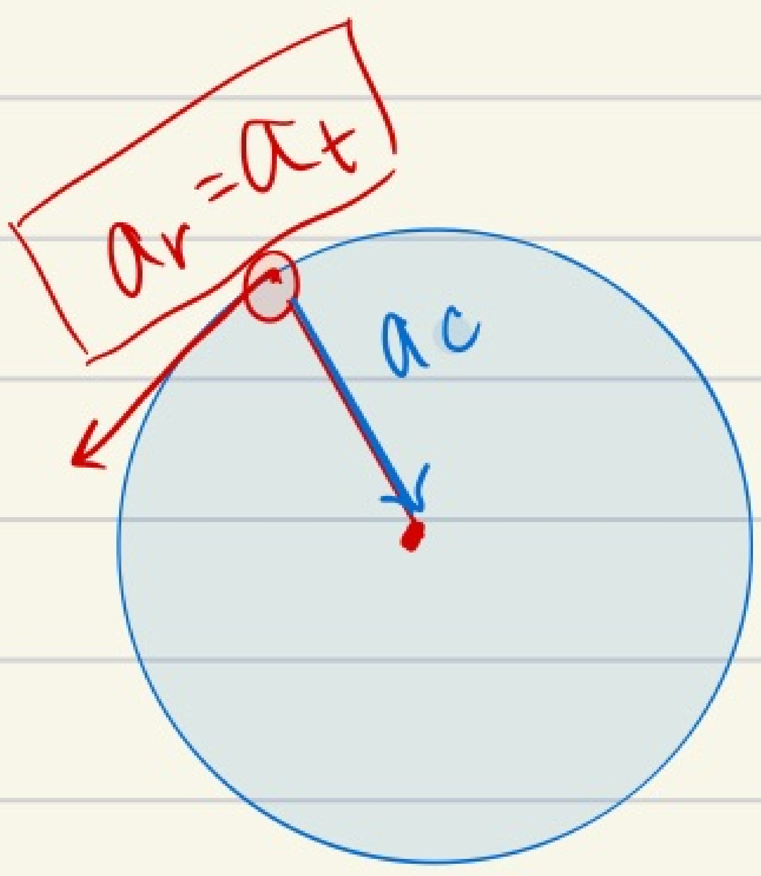
$$\boxed{v = r\omega}$$

linear speed

radius

angular vel.

t: tang
r: radial



$$r \cdot \alpha = \frac{\Delta\omega}{\Delta t} \cdot r = \frac{r(\omega_f - \omega_i)}{\Delta t} = \frac{\overbrace{v_f - v_i}^{\text{speed}}}{\Delta t} = a_{tan}$$

$$\boxed{a_{tan} = r\alpha}$$

The 4m long bar below starts from $\boxed{\text{rest}}$ and rotates through 5 revolutions with a constant angular acceleration of 30 rad/s^2.

a. How long did it take to make 5 revolutions?

b. What was the angular velocity after rotating 5 revolutions?

α

① $\omega_0 = 0$

$5 \text{ rev} = 5 \cdot 2\pi$

② $\Delta\theta = 10\pi \text{ rad}$

③ $\alpha = 30 \frac{\text{rad}}{s^2}$

$t = ?$

#3 $\Delta\theta = \overset{0}{\cancel{\omega_0}}\Delta t + \frac{1}{2}\alpha \Delta t^2$

$10\pi \text{ rad} = \frac{1}{2}\left(30 \frac{\text{rad}}{s^2}\right) \cdot \Delta t^2$

$\Delta t^2 = \frac{(10\pi \text{ rad})(2)}{30 \text{ rad}/s^2}$

$\Delta t = \sqrt{}$

$$\boxed{\Delta t = \frac{\sqrt{6}}{3} s = 1.45 \text{ s}}$$

ⓑ $\omega_f = ?$

$\omega_f = \omega_0 + \alpha \Delta t$

$\omega_f = 0 + \frac{30 \text{rad}}{s^2}(1.45 \text{ s})$

$$\boxed{\omega_f = 43.5 \frac{\text{rad}}{s}}$$

The 4m long bar below starts with an angular velocity of 40 rad/s and decelerates with a constant deceleration to a stop after rotating through 20 revolutions.

a. How fast is the edge of the bar moving initially in m/s?

b. What was the angular acceleration of the bar?

(a)

$\omega_i = 40 \text{ rad/s}$

$\omega_f = 0$

$\Delta\theta = 20 \cdot 2\pi \text{ rad}$
$= 40\pi \text{ rad}$

$v_i = r \cdot \omega_i =$
$(4m)(40 \text{ rad/s})$

$v_i = 160 \text{ m/s}$ or
200 m/s

(b). $\alpha = ?$

#5 $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

$0 = \omega_i^2 + 2\alpha\Delta\theta$

$0 = \left(\frac{40 \text{ rad}}{s}\right)^2 + 2(\alpha)(40\pi)$

$2 \cdot \alpha \cdot 40\pi = -(40)^2$

$\alpha = \frac{-(40)^2}{2 \cdot 40\pi} = -6.37 \frac{\text{rad}}{s^2}$

HW alg ch.8 1, 4, 7, 15, 16

calc ch.10 1, 4, 5, 16, 17

Rotational Kinematic Formulas

What do the Rotational Kinematic formulas mean?

The Rotational Kinematic formulas are the same 4 formulas we had for linear variables (Δx, v_i, v_f, a, t) but replaced with their angular counterparts ($\Delta\theta$, ω_i, ω_f, α, t).

4 Rotational Kinematic Formulas

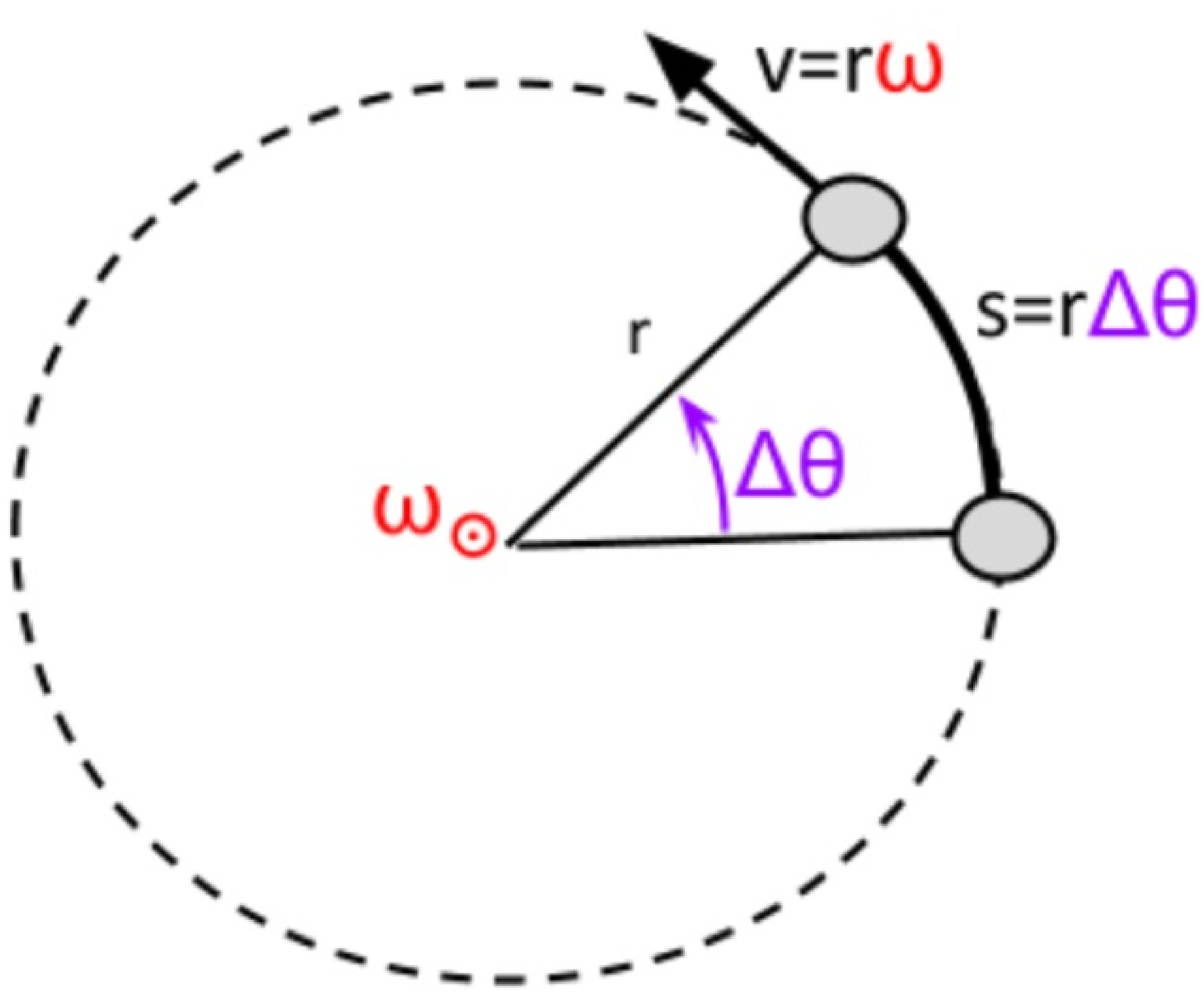
1. $\omega_f = \omega_i + \alpha t$
2. $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
3. $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
4. $(\omega_f + \omega_i)/2 = \Delta\theta / t$

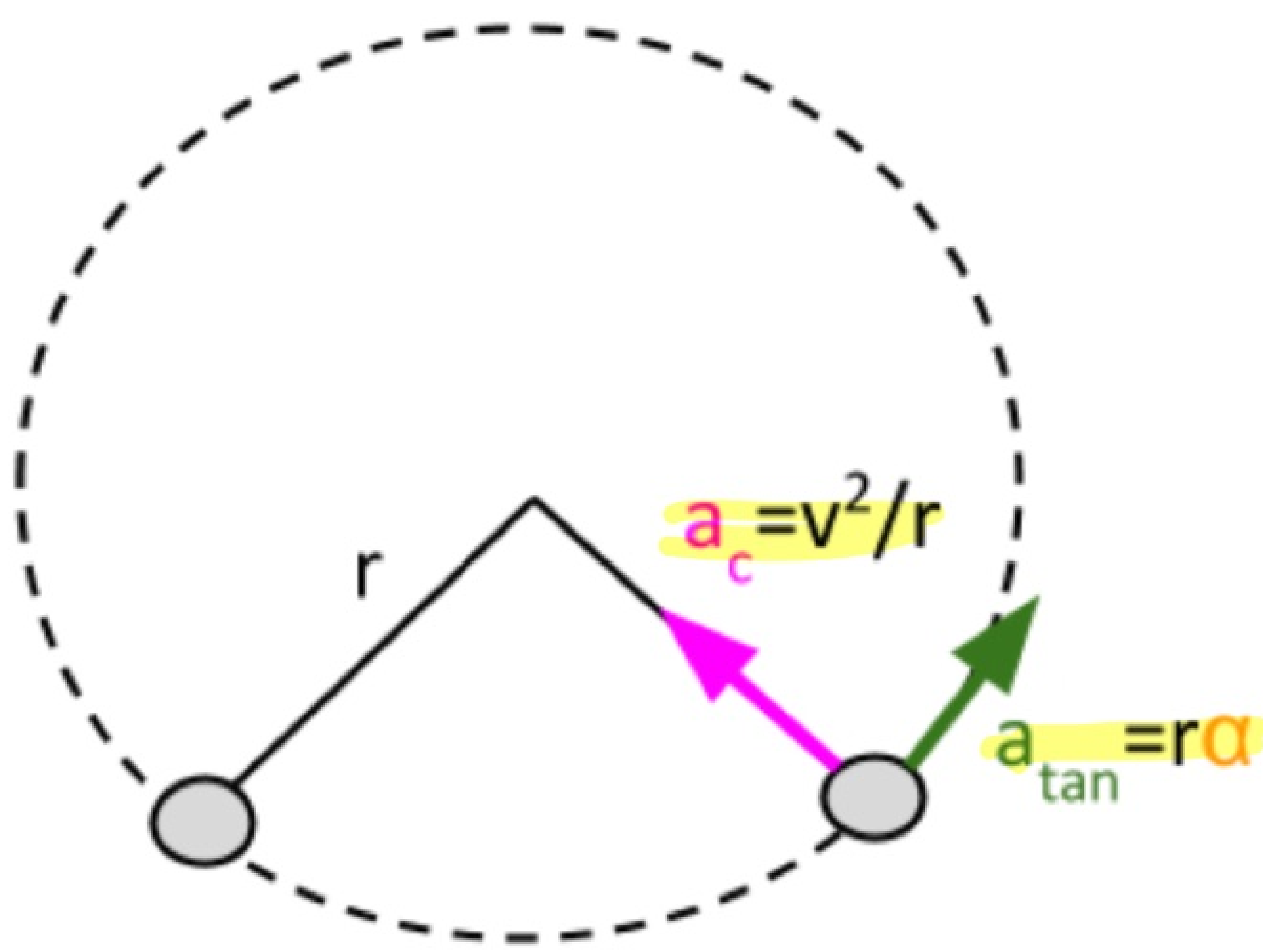
$x = \theta$

$v = \omega$

$a = \alpha$

Right hand rule to find direction of angular velocity ω.





Example Question:

Question: An object is rotating in a circle at a constant rate. Which best describes the accelerations of the object?

✓ direction

	Angular acc. α	**Tangential acc.** a_t	**Centripetal acc.** a_c
A.	Non-zero	Non-zero	Zero
B.	Zero	Zero	Zero
C.	Non-zero	Non-zero	Non-zero
D.	Zero	Zero	Non-zero

Torque τ

Units: **Nm** **Vector?** **Yes**

What does torque mean?

Force is what causes acceleration. Torque is what causes angular acceleration.

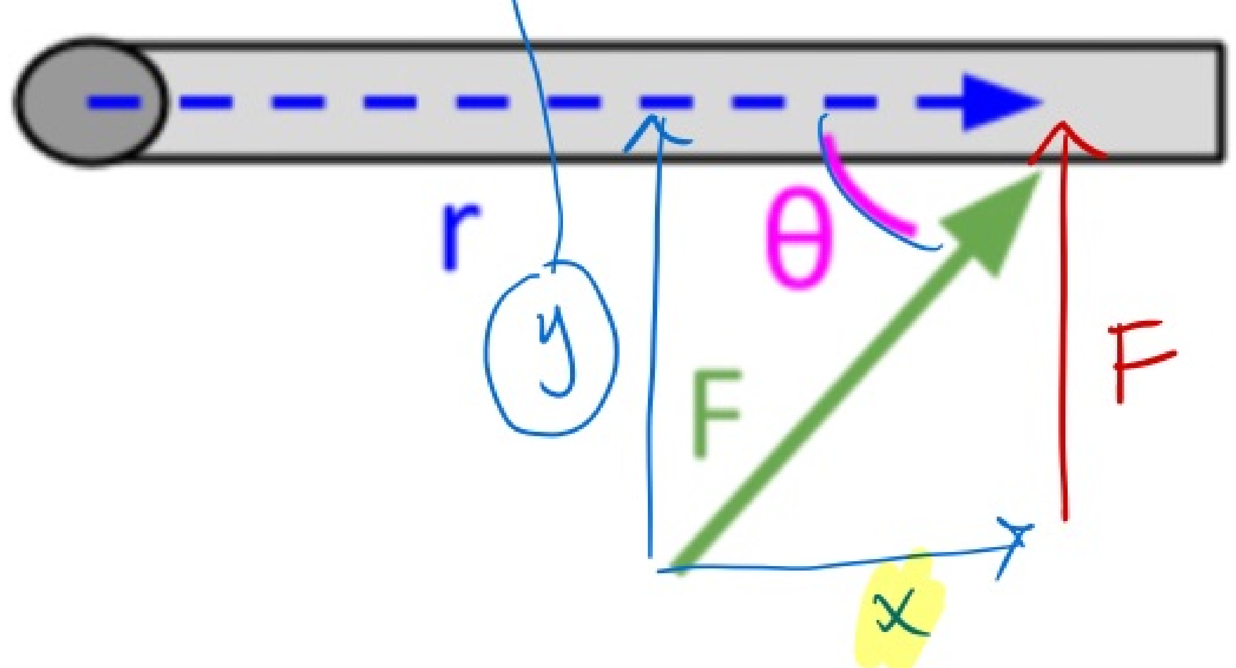
$$\tau = rF\boxed{\sin\theta}$$

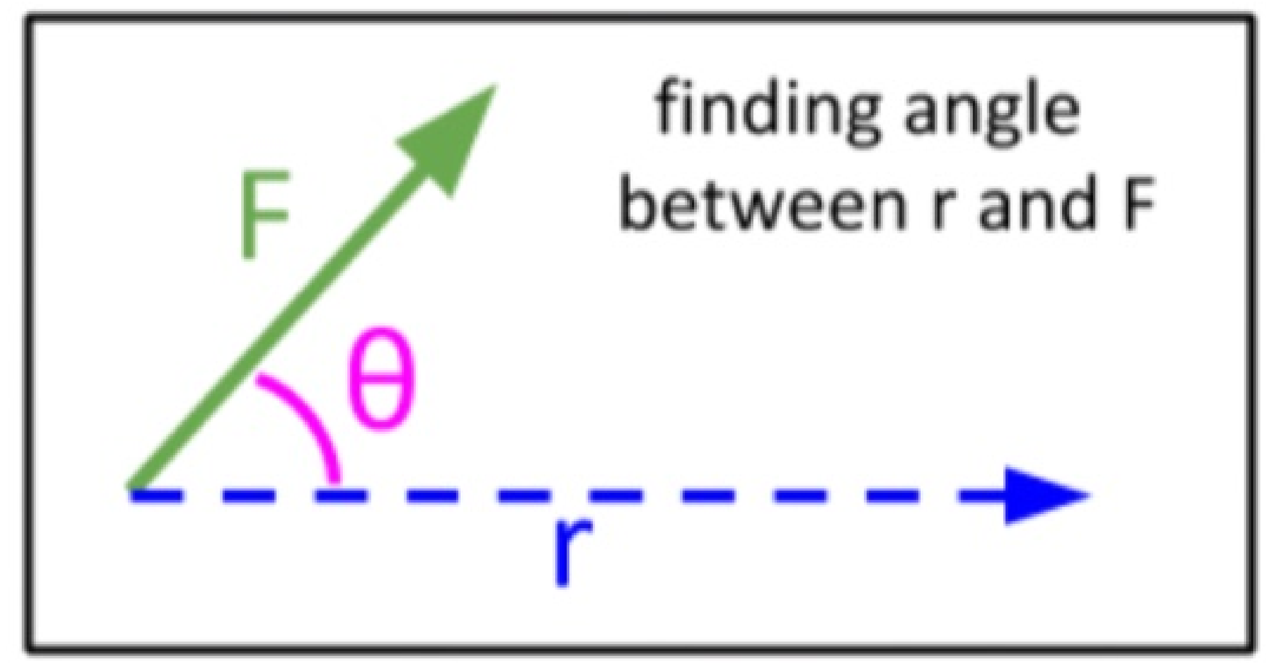
$\theta = 90°$

$\sin\theta = 1$

$\tau = rF$

τ



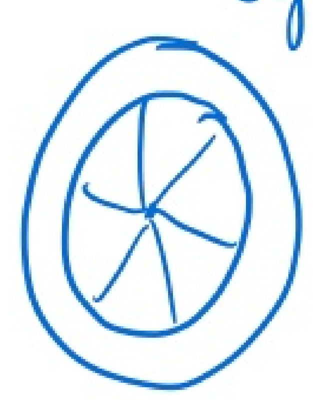


Conditions for equilibrium

"Translational" Equilibrium: $\Sigma F = 0$

"Rotational" Equilibrium: $\Sigma \tau = 0$

cylinder



I moment of inertia

$\tau = r \cdot F \cdot \sin\theta$

$F = m \cdot a$

$\tau = I \cdot \alpha$

Example Question:

Question: A ball is rotating in a circle and slowing down. In reference to the direction of the ball's velocity v and angular velocity ω, what are the directions of the net torque, tangential force, and centripetal force on the ball?

	Net torque	Tangential force	Centripetal force
A.	Opposite to ω	no direction (zero)	perpendicular to v
B.	Same direction as ω	Opposite to v	no direction (zero)
C.	Opposite to ω	Opposite to v	perpendicular to v
D.	Perpendicular to ω	no direction (zero)	no direction (zero)

EXAMPLE 10–7 **Torque on a compound wheel.** Two thin disk-shaped wheels, of radii $R_A = 30\,\text{cm}$ and $R_B = 50\,\text{cm}$, are attached to each other on an axle that passes through the center of each, as shown in Fig. 10–15. Calculate the net torque on this compound wheel due to the two forces shown, each of magnitude 50 N.

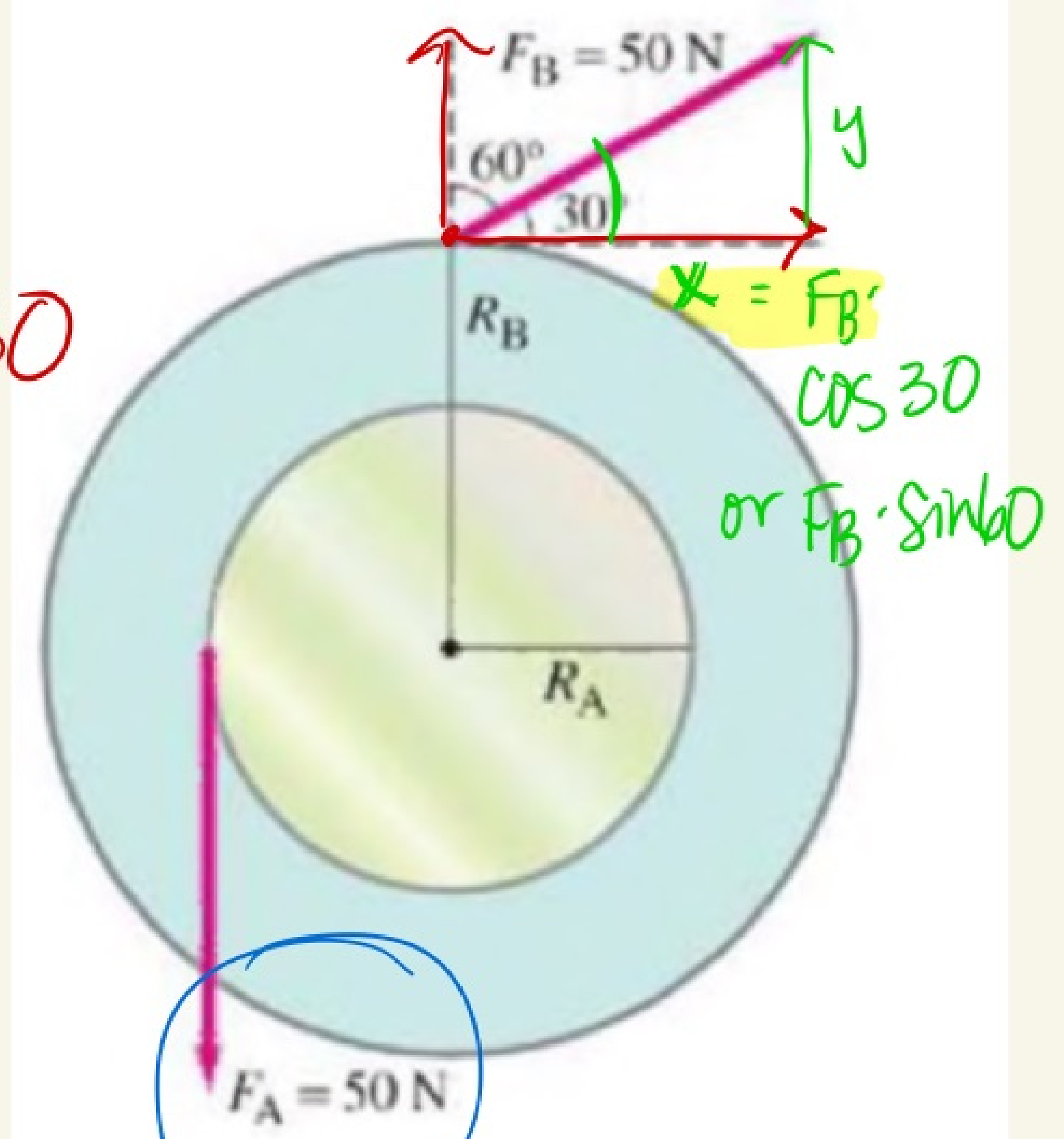
$\tau = I\alpha$

$m \cdot N$

CCW (+)

F_A (+)

F_B (−)



x = $F_B \cdot \cos 30$ or $F_B \cdot \sin 60$

FIGURE 10–15 Example 10–7. The torque due to $\vec{F}_A$ tends to accelerate the wheel counterclockwise, whereas the torque due to $\vec{F}_B$ tends to accelerate the wheel clockwise.

$\tau_B = R_B \cdot F_B \cdot \cos 30$

$\tau_A = r \cdot F \cdot \sin\theta$

$= R_A \cdot F_A \sin 90$

$\tau_A = R_A \cdot F_A$

$\Sigma\tau = \tau_A + \tau_B = R_A \cdot F_A - R_B F_B \cos 30$

$\tau_{Net} = (0.3\,m)(50N) - (0.5\,m)(50N)\cos 30$

$\boxed{\tau_{Net} = -6.7\,mN}$

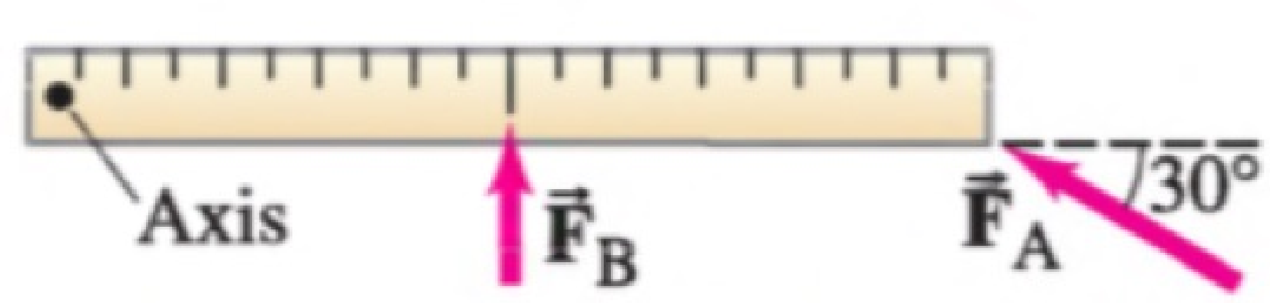
EXERCISE B Two forces ($F_B = 20\,\text{N}$ and $F_A = 30\,\text{N}$) are applied to a meter stick which can rotate about its left end, Fig. 10–16. Force $\vec{F}_B$ is applied perpendicularly at the midpoint. Which force exerts the greater torque: F_A, F_B, or both the same?

$\tau = r \cdot F_{\perp}$

$\tau_A = 15\,mN$

$\tau_B = 10\,mN$

FIGURE 10–16 Exercise B.



Rotational Inertia I

Units: kg m^2 **Vector?** **No**

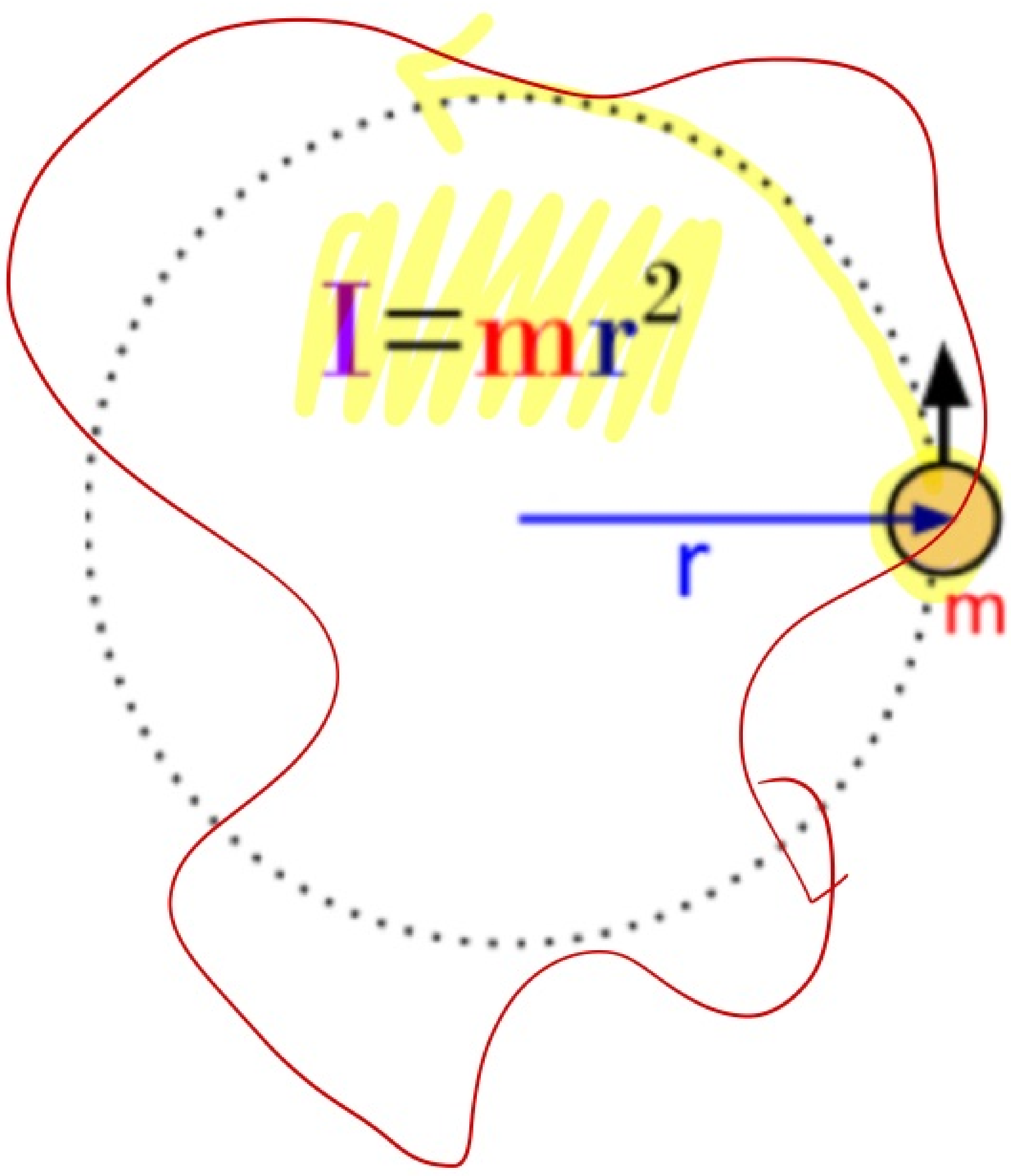
What does Rotational Inertia mean?

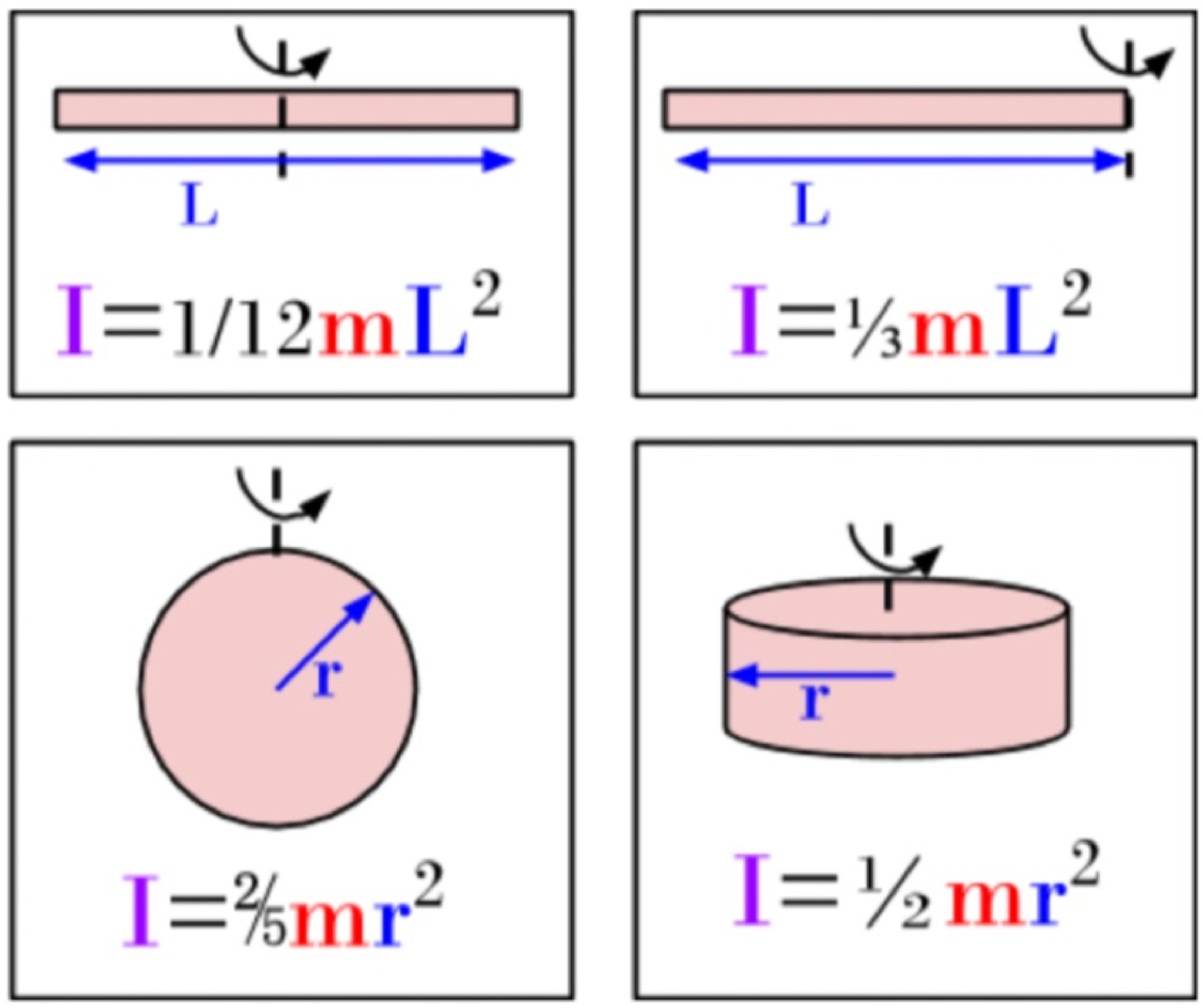
An object with a larger Rotational inertia will be harder to get rotating, and harder to stop rotating. Rotational inertia is also called "Moment of inertia".

An object will have a larger rotational inertia if its mass is distributed far from **the axis**.
An object will have a smaller rotational inertia if its mass is distributed close to **the axis**.

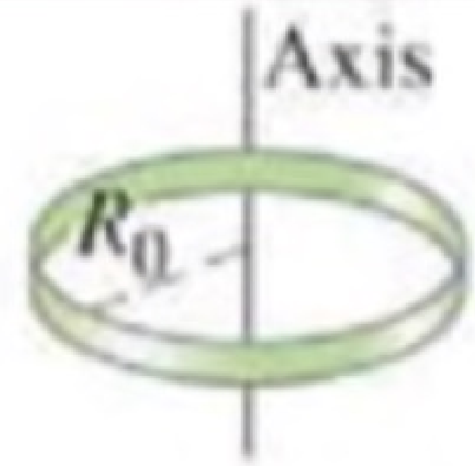
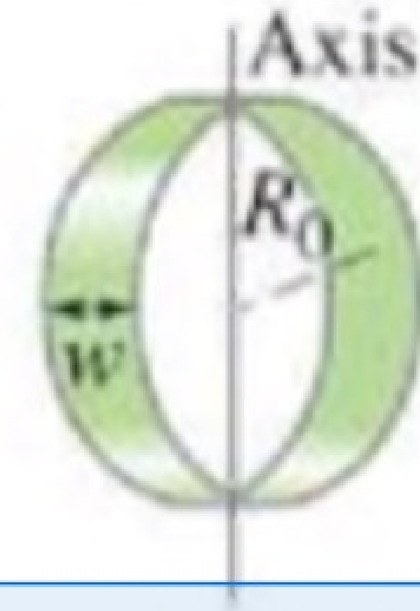
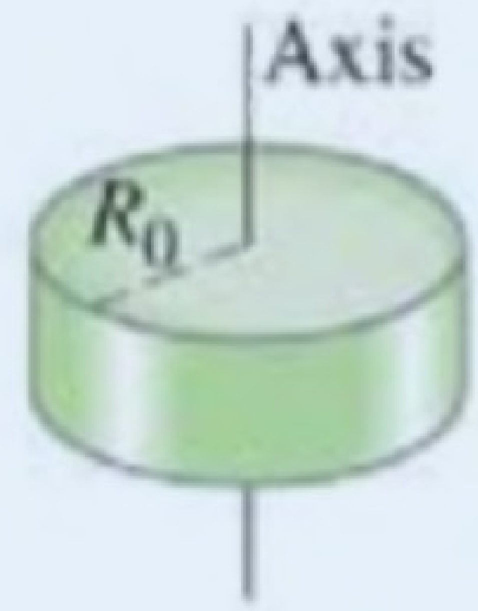
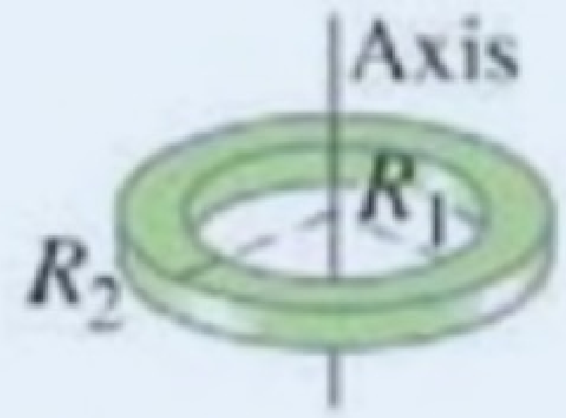
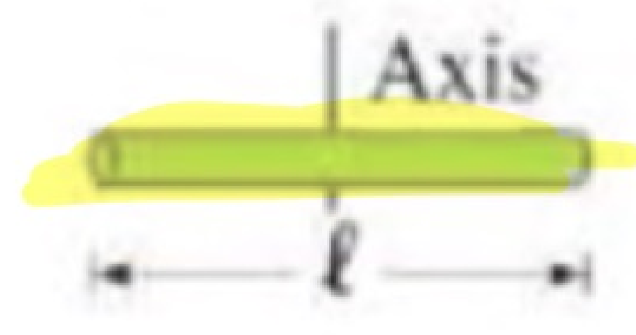
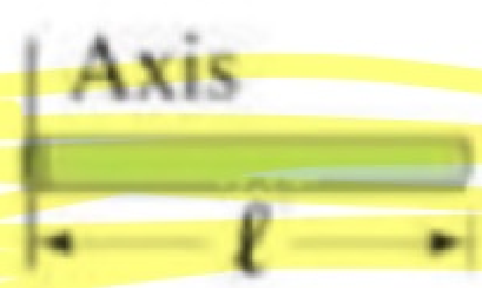
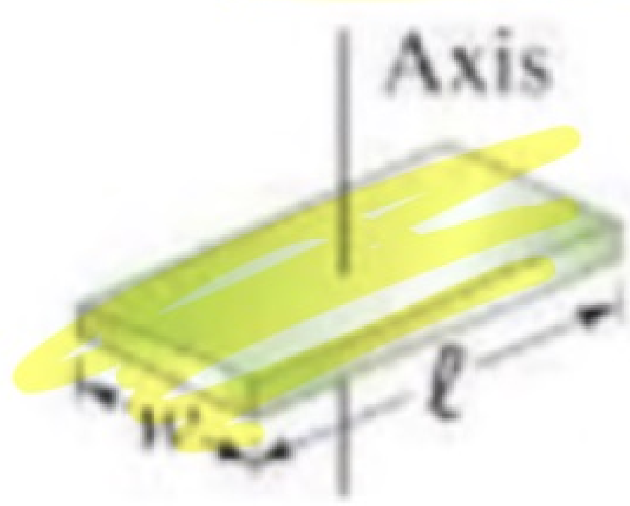
$I = mr^2$ (single mass going in a circle of a single radius)

$I = \Sigma mr^2$ (multiple individual masses going in circles of different radius)





Rotational Inertia = Moment of Inertia

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R_0	Through center		MR_0^2
(b)	**Thin hoop,** radius R_0 width w	Through central diameter		$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder,** radius R_0	Through center		$\frac{1}{2}MR_0^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius r_0	Through center		$\frac{2}{5}Mr_0^2$
(f)	**Long uniform rod,** length ℓ	Through center		$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod,** length ℓ	Through end		$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate,** length ℓ, width w	Through center		$\frac{1}{12}M(\ell^2 + w^2)$

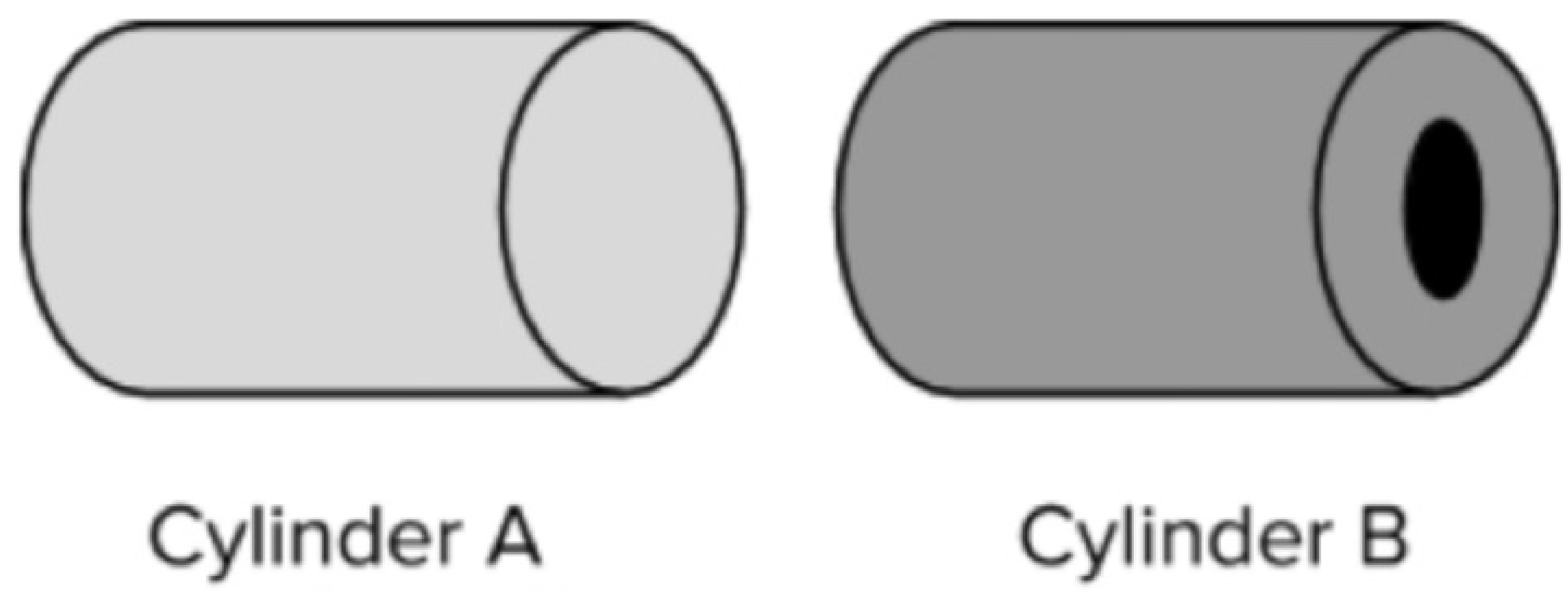
#1 Add to your HW →

why does the spinning wheel move Olivia?

Example Question:

Question: Two cylinders are allowed to roll without slipping down a hill from rest. The mass of cylinder A is distributed evenly throughout the cylinder. Cylinder B is made from a more dense material and has a hollow center with the mass surrounding the central axis as seen in the diagram below. The masses and radii of each cylinder are the same. Which cylinder will reach the bottom of the hill first?

A. Cylinder A
B. Cylinder B
C. They tie
D. The densities are needed to say



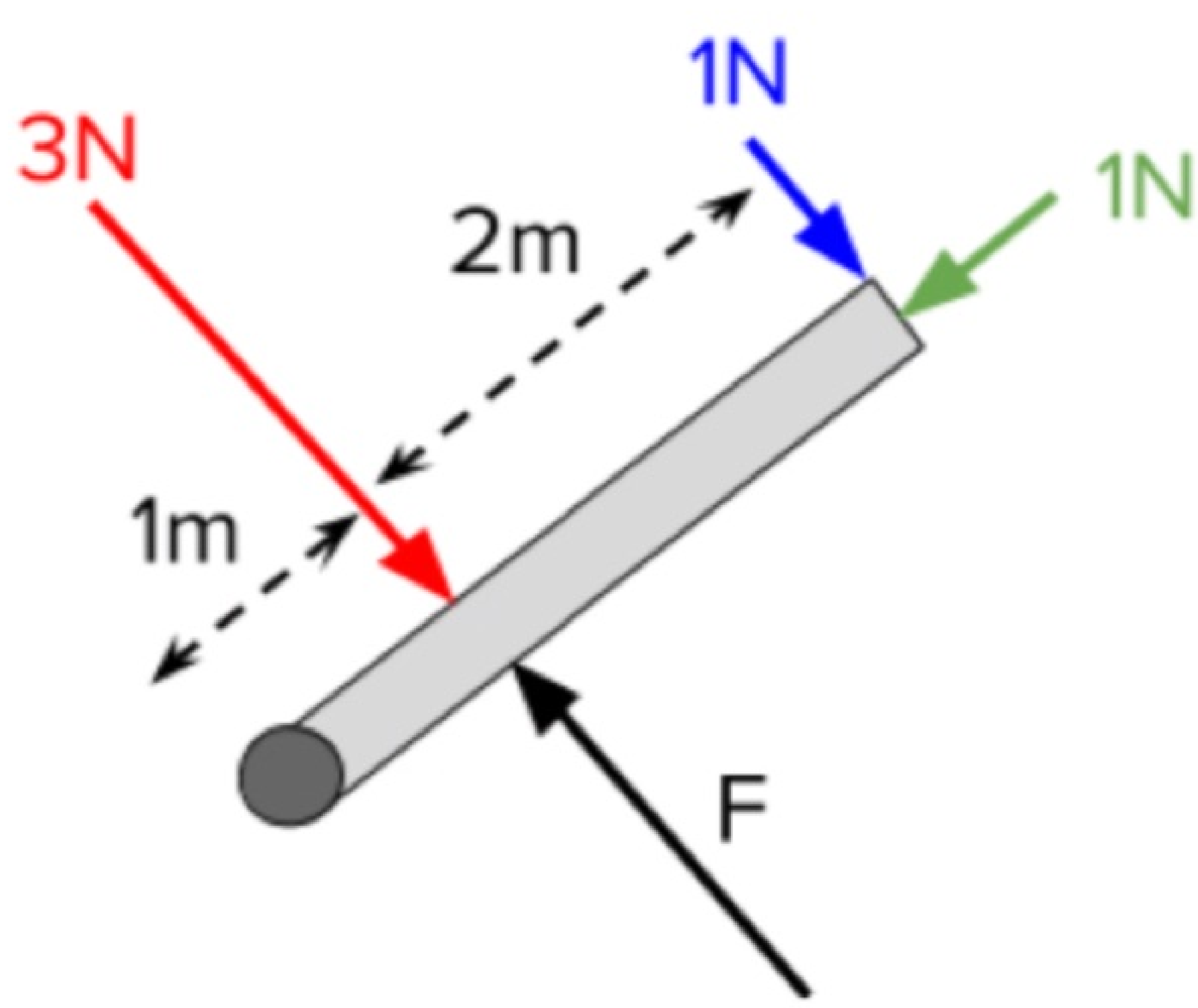
[redacted]

#2

Example Question:

Question: Different forces are applied to a rod which can rotate about an axis at its end. How large would the force **F** have to be in order for the rod to be in rotational equilibrium?

A. 3N
B. 4N
C. 6N
D. 10N



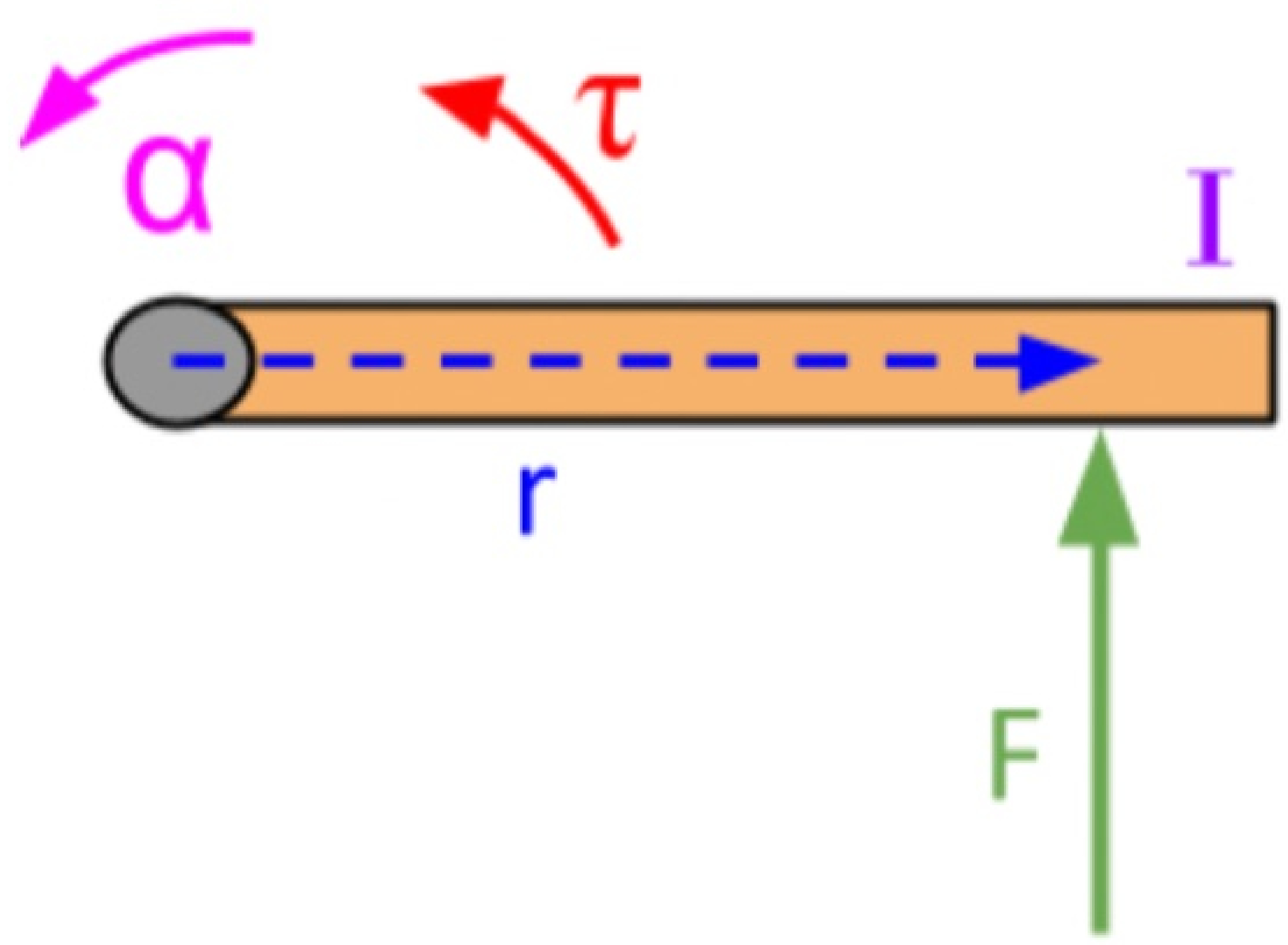
[redacted]

Angular version of Newton’s Second Law

What does the Angular version of Newton’s Second Law mean?

The angular version of Newton’s Second Law says that the **angular acceleration** is proportional to the **net torque**, and inversely proportional to the **rotational inertia**.

$\alpha = \Sigma\tau/I$

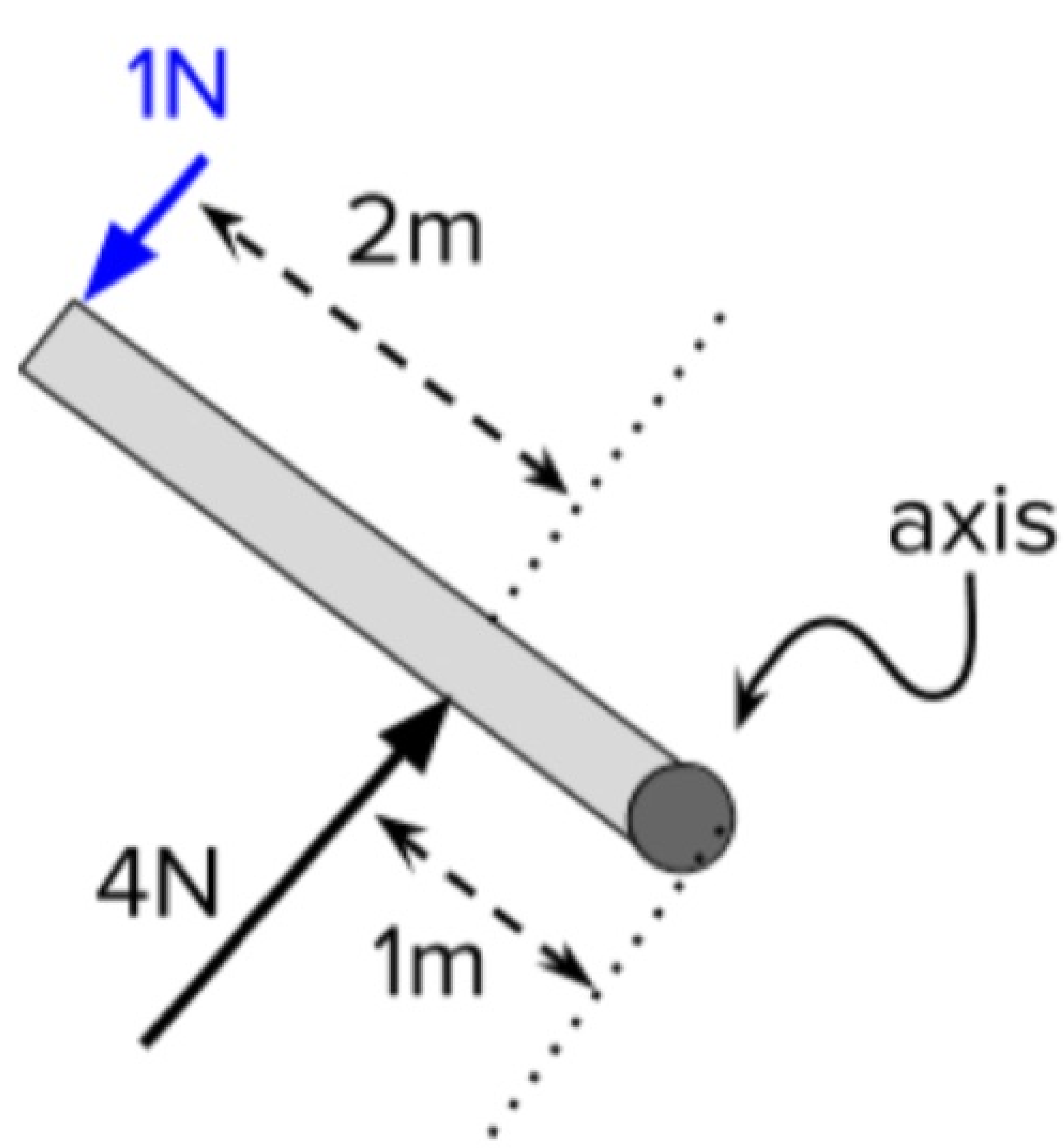


Warning: Torque is a vector, so it can be positive (CCW) or negative (CW).

Example Question:

Question: The rod shown below has a rotational inertia of 2 kg m^2 and the forces acting on it as shown. What is the magnitude of the angular acceleration of the rod?

A. 0.5 rad/s^2
B. 1.0 rad/s^2
C. 1.5 rad/s^2
D. 2.0 rad/s^2



[redacted]

Rotational Kinetic Energy Units: kg m^2/s^2 Vector? **No**

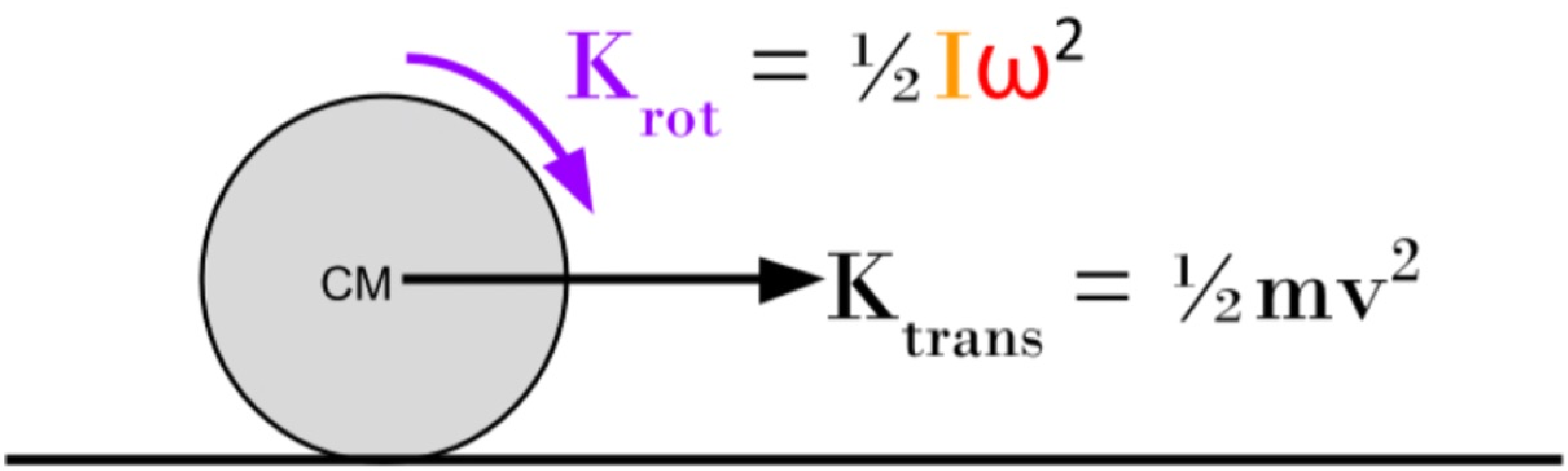
What does Rotational Kinetic Energy mean?

If an object is rotating it has rotational kinetic energy.

If the center of mass of the object is moving, and the object is rotating, it will have regular **translational kinetic energy** and rotational kinetic energy.

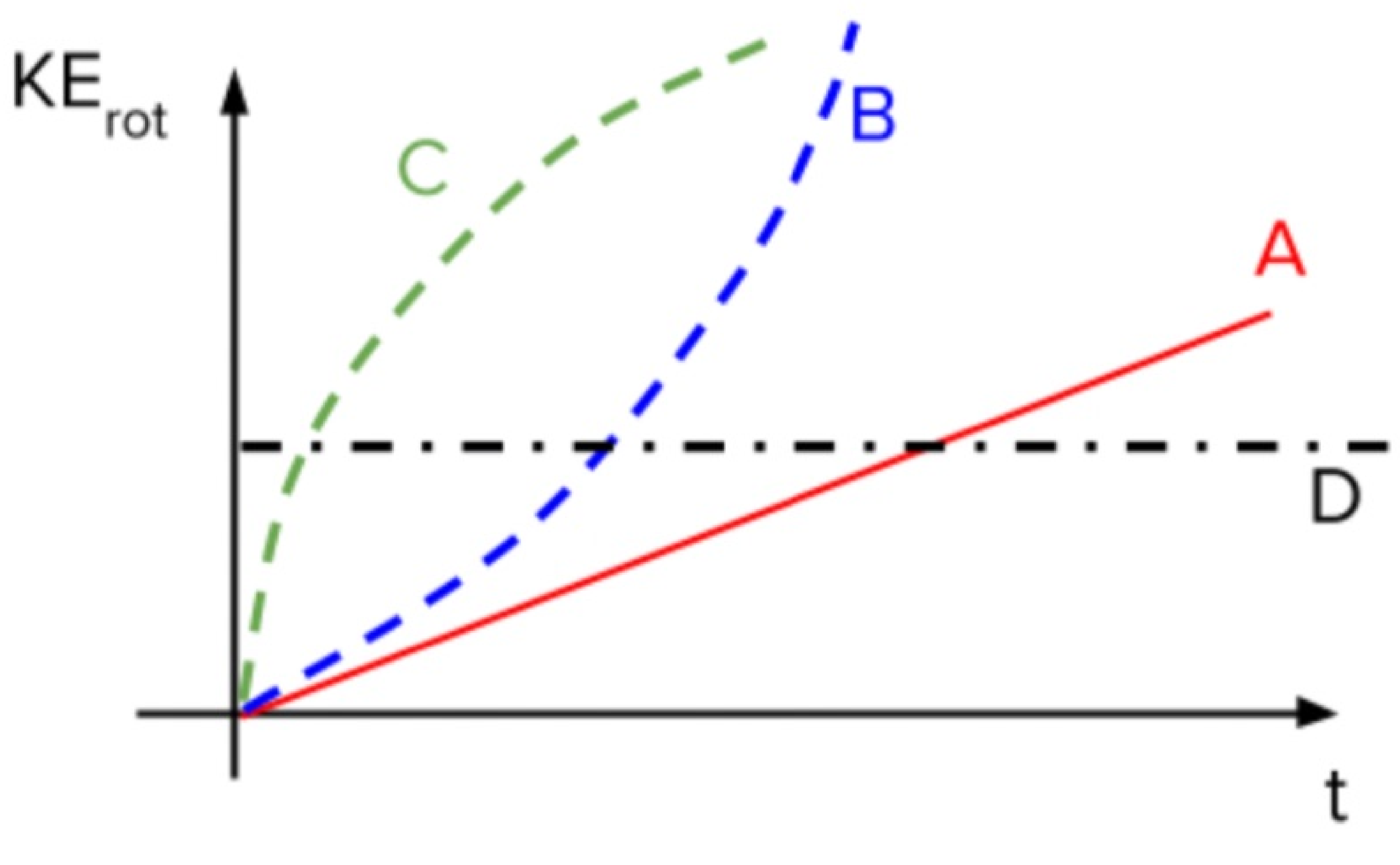
$K_{rotational} = \frac{1}{2} I \omega^2$ (if the object is rotating with angular velocity ω)

$K_{translational} = \frac{1}{2} m v^2$ (if the center of mass of the object is moving with speed v)



Example Question:

Question: A constant torque is exerted on a cylinder that is initially at rest which can rotate about an axis through its center. Which curve best gives the rotational kinetic energy of the cylinder as a function of time?



A. A
B. B
C. C
D. D

Answer: B

θ : dis
ω : vel.
α : acc

Linear Momentum

$p = m \cdot v$

Angular mom $\Longrightarrow$ $\boxed{L = I \cdot \omega}$

linear
$\Delta p = m_f v_f - m_i v_i = J = F \cdot \Delta t$

Angular
$\Delta L = I_f \omega_f - I_i \omega_i = \tau \cdot \Delta t$

Linear
$KE_L = \frac{1}{2} m v^2$

rotational
$KE_R = \frac{1}{2} I \omega^2$

I is dependent on shape

$KE_s = KE_L + KE_R$

$KE_s = KE_L + KE_R$

Angular Momentum

Units: **kg m²/s** Vector? **Yes**

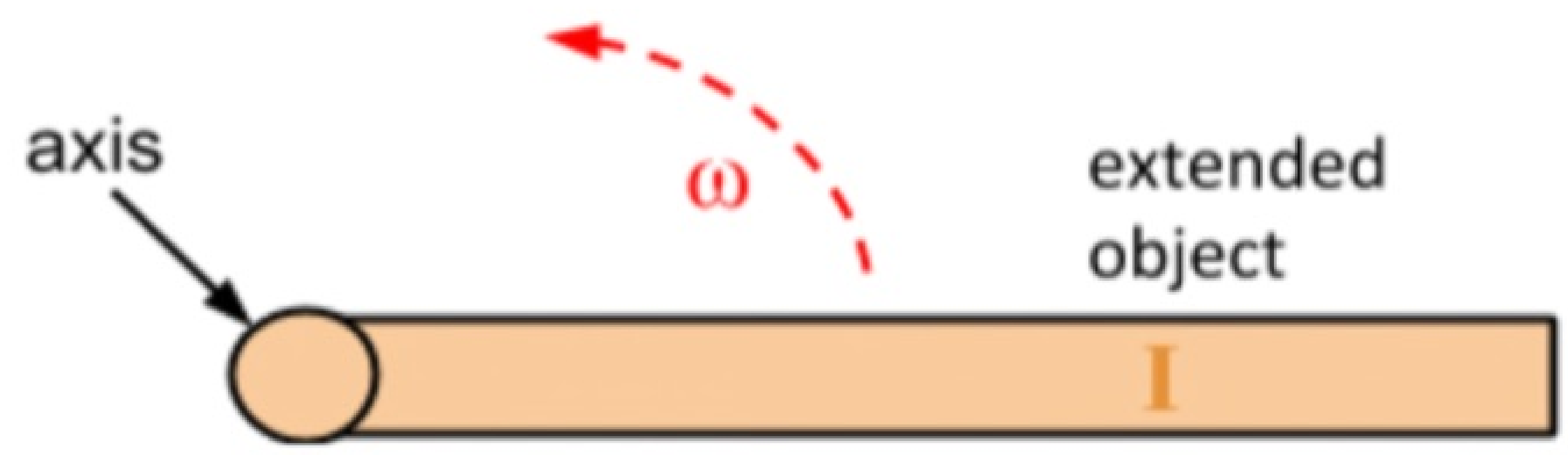
What does Angular Momentum mean?

Angular momentum is conserved if there is **no external torque**.

Even a point mass moving in a straight line can have angular momentum (since if it hits something it can cause that thing to start rotating.)

$L = I\omega$ (extended objects)

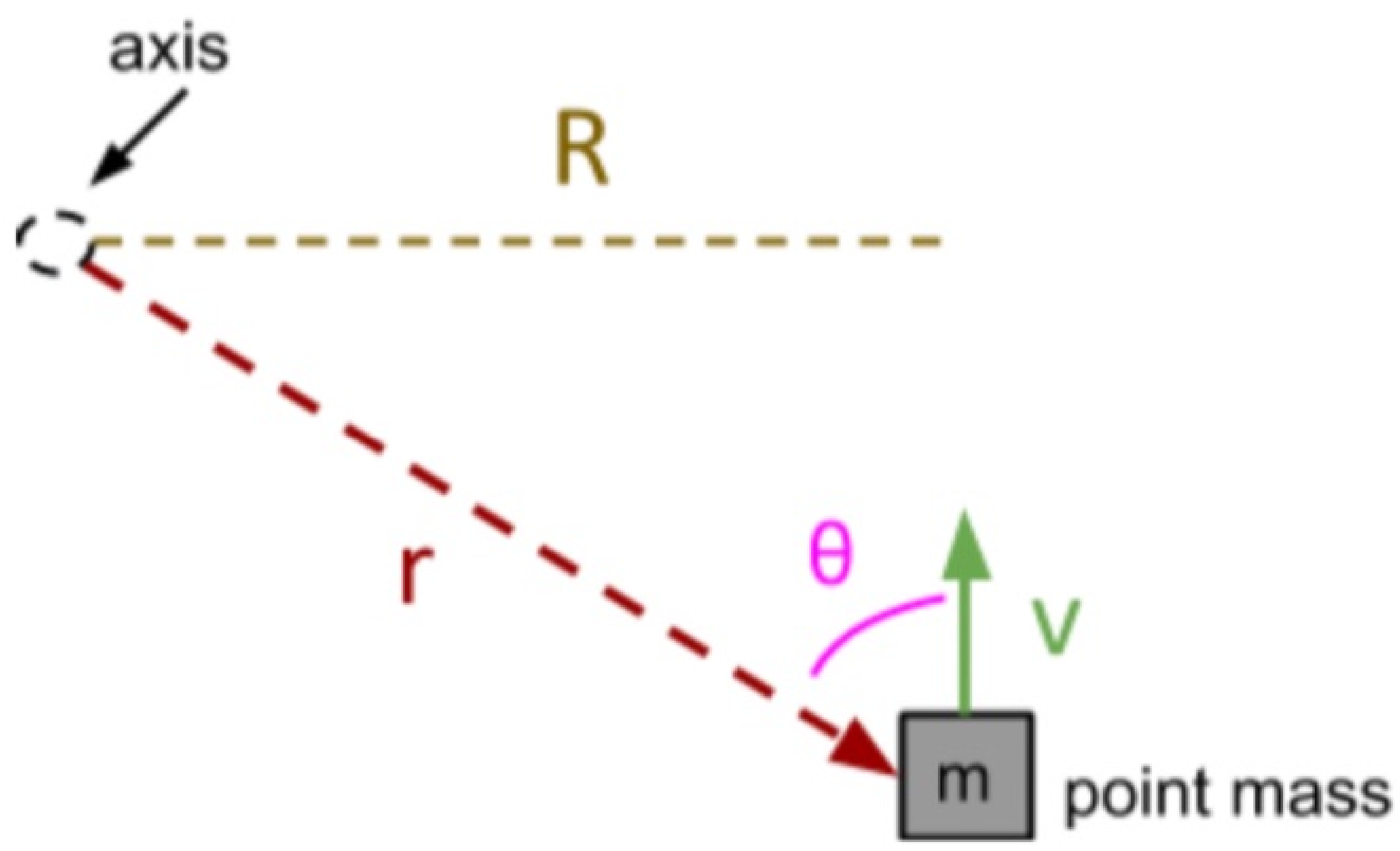
L = angular momentum
I = rotational inertia
ω = angular velocity



$L = mv(r\sin\theta)$ (point masses)

$L = mv(R)$

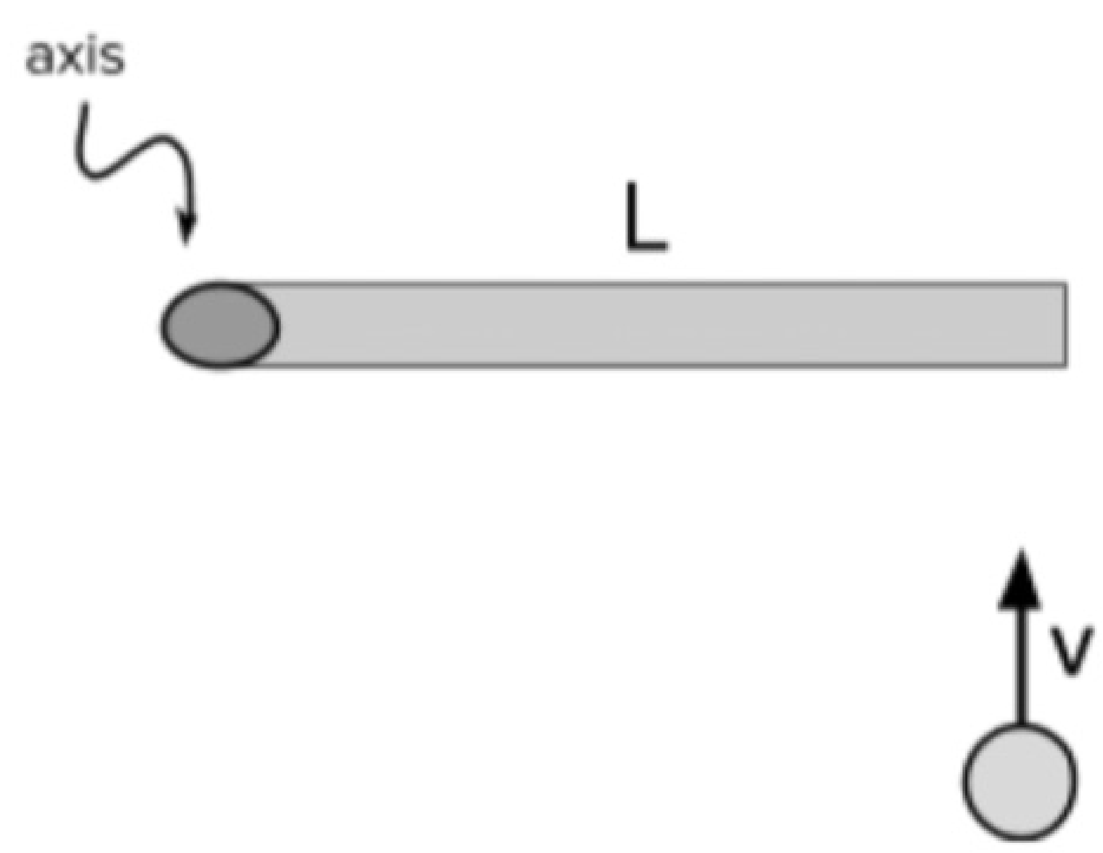
L = angular momentum
m = mass moving with speed v
v = velocity of the mass
r = distance from the axis to the mass m
θ = angle between r and the velocity of the mass
P = point of closest approach, which is equal to $r\sin\theta$



Example Question:

Question: A clay sphere of mass M is heading toward a rod of mass 3M and length L with a speed v. The rod is free to rotate about an axis at its end. If the clay slicks to the end of the rod, what is the angular velocity of the rod after the clay sticks to the rod? (The moment of inertia of a rod about its end is $\frac{1}{3}mL^2$)

A. $\frac{v}{2L}$
B. $\frac{2v}{L}$
C. $\frac{3v}{2L}$
D. $\frac{2v}{3L}$



Answer: A

EXAMPLE 8–6 **Centrifuge acceleration.** A centrifuge rotor is accelerated from rest to 20,000 rpm in 30 s. (*a*) What is its average angular acceleration? (*b*) Through how many revolutions has the centrifuge rotor turned during its acceleration period, assuming constant angular acceleration?

$\alpha = \frac{\Delta \omega}{\Delta t}$

$\omega = 2\pi f$

a

20,000 rpm in 30 s

$$\omega = 2\pi \frac{\text{rad}}{\cancel{\text{rev}}} \cdot \frac{20{,}000 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}}{60 \frac{\text{s}}{\cancel{\text{min}}}} = 2100 \frac{\text{rad}}{\text{s}}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{2100\ \text{rad/s}}{30\ \text{s}} = 70 \frac{\text{rad}}{\text{s}^2}$$

b

$$\theta = 0 + \tfrac{1}{2}(70\ \text{rad/s}^2)(30\ \text{s})^2 = 3.15 \times 10^4\ \text{rad}$$

$$\frac{3.15 \times 10^4\ \cancel{\text{rad}}}{2\pi\ \text{rad/rev}} = 5000\ \text{rev}$$

HW Due Wed. : Calc ch. 10 89, 94 ch. 11 3, 10, 78

Alg. ch. 8 53, 73, 80, 84, 85

EXAMPLE 11–2 **Clutch.** A simple clutch consists of two cylindrical plates that can be pressed together to connect two sections of an axle, as needed, in a piece of machinery. The two plates have masses $M_A = 6.0\,\text{kg}$ and $M_B = 9.0\,\text{kg}$, with equal radii $R_0 = 0.60\,\text{m}$. They are initially separated (Fig. 11–4). Plate M_A is accelerated from rest to an angular velocity $\omega_1 = 7.2\,\text{rad/s}$ in time $\Delta t = 2.0\,\text{s}$. Calculate (*a*) the angular momentum of M_A, and (*b*) the torque required to have accelerated M_A from rest to ω_1. (*c*) Next, plate M_B, initially at rest but free to rotate without friction, is placed in firm contact with freely rotating plate M_A, and the two plates both rotate at a constant angular velocity ω_2, which is considerably less than ω_1. Why does this happen, and what is ω_2?

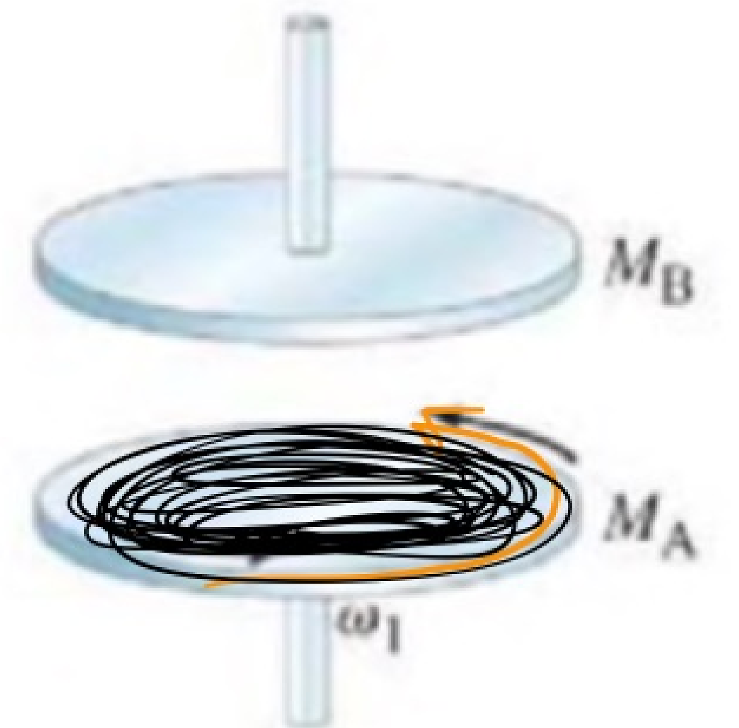


FIGURE 11–4 Example 11–2.

$M_a = 6.0\,\text{kg}$ $\quad M_B = 9.0\,\text{kg}$

$R_a = 0.6\,\text{m}$ $\quad R_B = 0.6\,\text{m}$

$L_A = I_A \cdot \omega_A$ $\qquad I_A = \frac{1}{2} m_A R_A^2$

$L_A = \left(\frac{1}{2} m_A R_A^2\right)(7.2\,\text{rad/s})$

$L_A = \frac{1}{2}(6.0\,\text{kg})(0.6\,\text{m})^2(7.2\,\text{rad/s})$

a) $L_A = 7.8\ \text{kgm}^2/\text{s}$

Angular Eqns.

$L = I\omega$ momentum $\qquad I$: Rot. inertia

$\tau = I \cdot \alpha$

$\Delta L = \tau \cdot \Delta t$ impulse

$F = m \cdot a$

τ Rot Force

α: Rot. acc ang acc.

$\Delta p = J = \boxed{F \cdot \Delta t}$

$\tau = ?$ $\qquad M_A$ from Rest to ω_1

$\Delta L = \tau \cdot \Delta t$

$7.8\,\frac{\text{kgm}^2}{\text{s}} = \tau \cdot \Delta t$

$\Delta L = L_f - \cancel{L_i}^{\,0}$

$\tau = \frac{7.8\,\frac{\text{kgm}^2}{\text{s}}}{2.0\,\text{s}}$ $\qquad \Delta t = 2.0\,\text{s}$

$\omega_2 = ?$

$\tau = 3.9\ \text{mN}$

$L_i = I_A \omega_1$ $\qquad L_f = (I_A + I_B)\omega_2$

$\omega_2 = \frac{I_A \omega_1}{I_A + I_B}$

$L_i = L_f$

$I_A \omega_1 = (I_A + I_B)\omega_2$

$\omega_2 = \frac{7.8\,\text{kgm}^2/\text{s}}{\frac{1}{2}(6.0\,\text{kg})(0.6\,\text{m})^2 + \frac{1}{2}(9.0\,\text{kg})(0.6\,\text{m})^2} = 2.7\,\frac{\text{rad}}{\text{s}}$

Bicycle gears: (*a*) How is the angular velocity ω_R of the rear wheel of a bicycle related to the angular velocity ω_F of the front sprocket and pedals? Let N_F and N_R be the number of teeth on the front and rear sprockets, respectively, Fig. 10–64. The teeth are spaced the same on both sprockets and the rear sprocket is firmly attached to the rear wheel. (*b*) Evaluate the ratio ω_R/ω_F when the front and rear sprockets have 52 and 13 teeth, respectively, and (*c*) when they have 42 and 28 teeth.

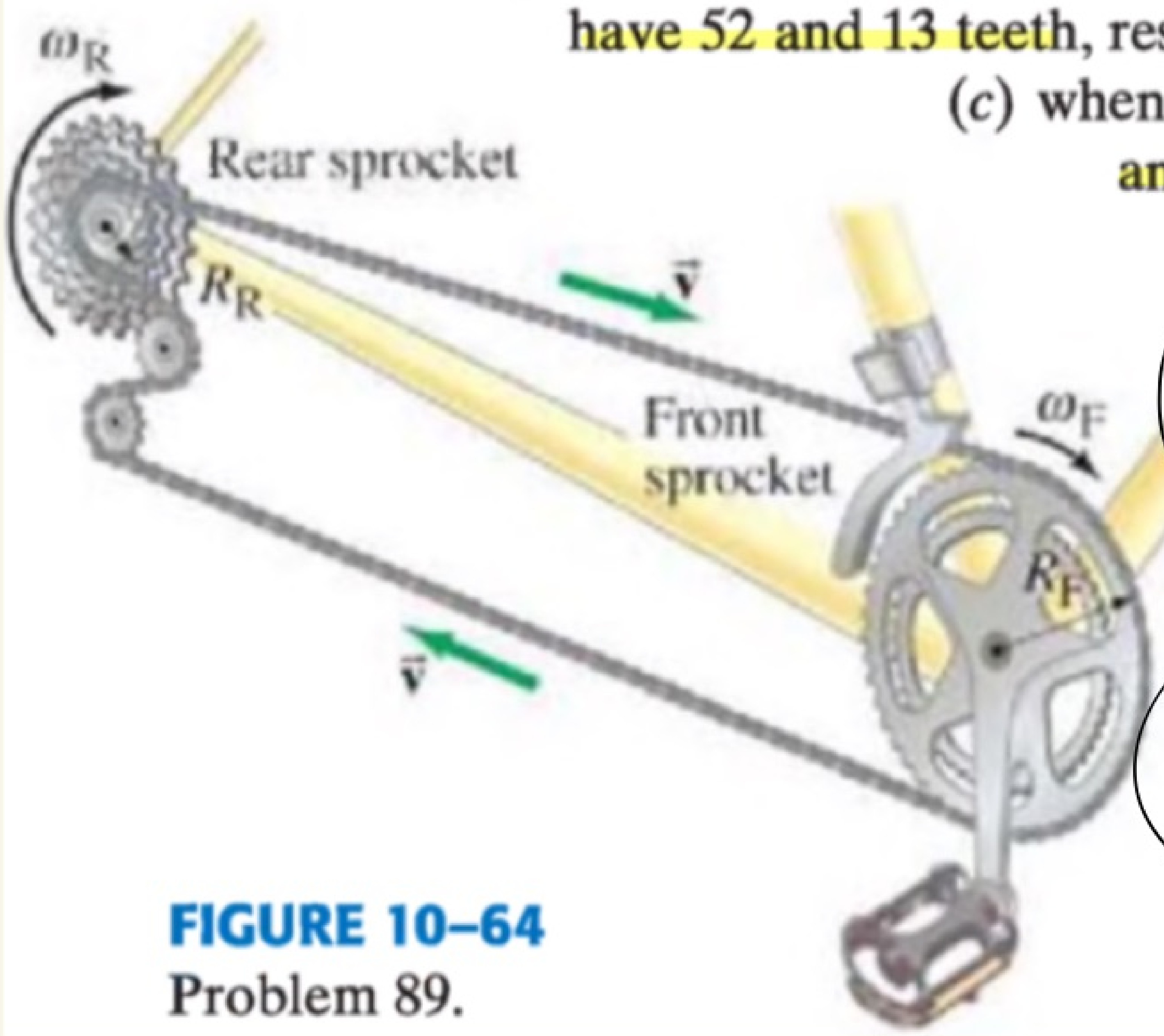


FIGURE 10–64
Problem 89.

linear speed$_R$ = linear speed$_F$

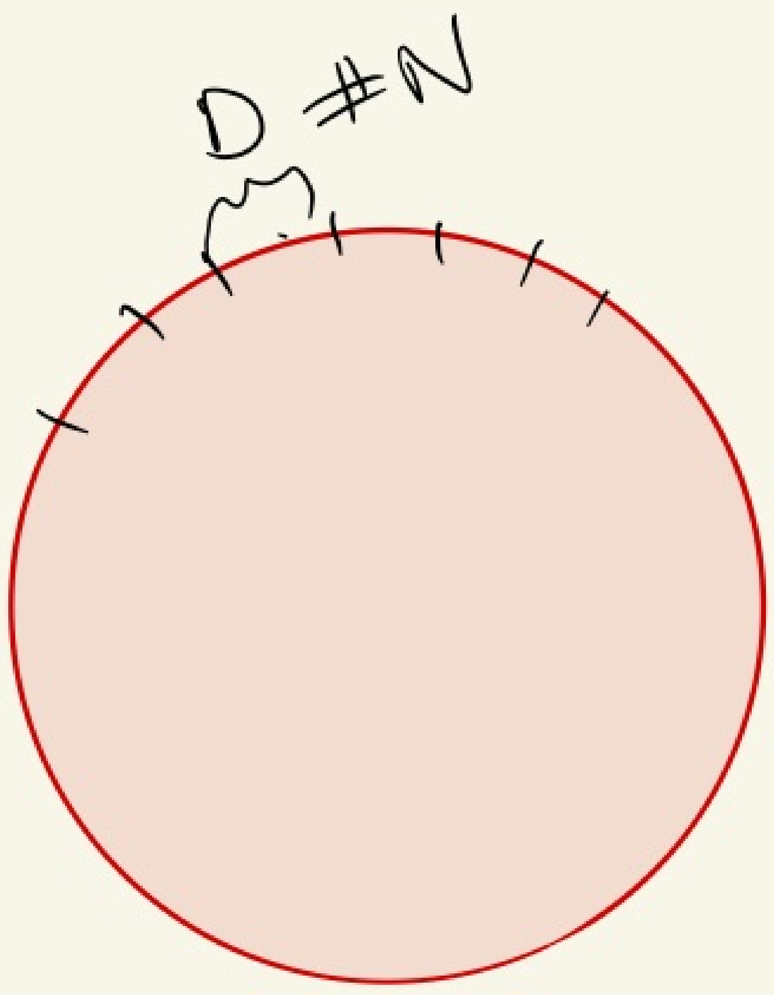
$N_R = 52$
$N_F = 13$

$N_R = 42$
$N_F = 28$

$V = r\omega$

$V_R = V_F$

$r_R \omega_R = r_F \omega_F$



Number of teeth

N: # teeth

D: distance

$C = 2\pi r$

$\underbrace{N \cdot d} = \underbrace{2\pi r}$

$r = \frac{Nd}{2\pi}$

$\frac{\omega_R}{\omega_F}$

$\omega_R \frac{N_R \cancel{D}}{\cancel{2\pi}} = \omega_F \frac{N_F \cancel{d}}{\cancel{2\pi}} \Rightarrow \frac{\omega_R}{\omega_F} = \frac{N_F}{N_R}$

(A)

$\frac{\omega_R}{\omega_F} = \frac{52}{13} = 4.0$

(B) $\frac{\omega_R}{\omega_F} = \frac{42}{28} = 1.5$

A marble of mass m and radius r rolls along the looped rough track of Fig. 10–67. What is the minimum value of the vertical height h that the marble must drop if it is to reach the highest point of the loop without leaving the track? (*a*) Assume $r \ll R$; (*b*) do not make this assumption. Ignore frictional losses.

$PE = mgh$

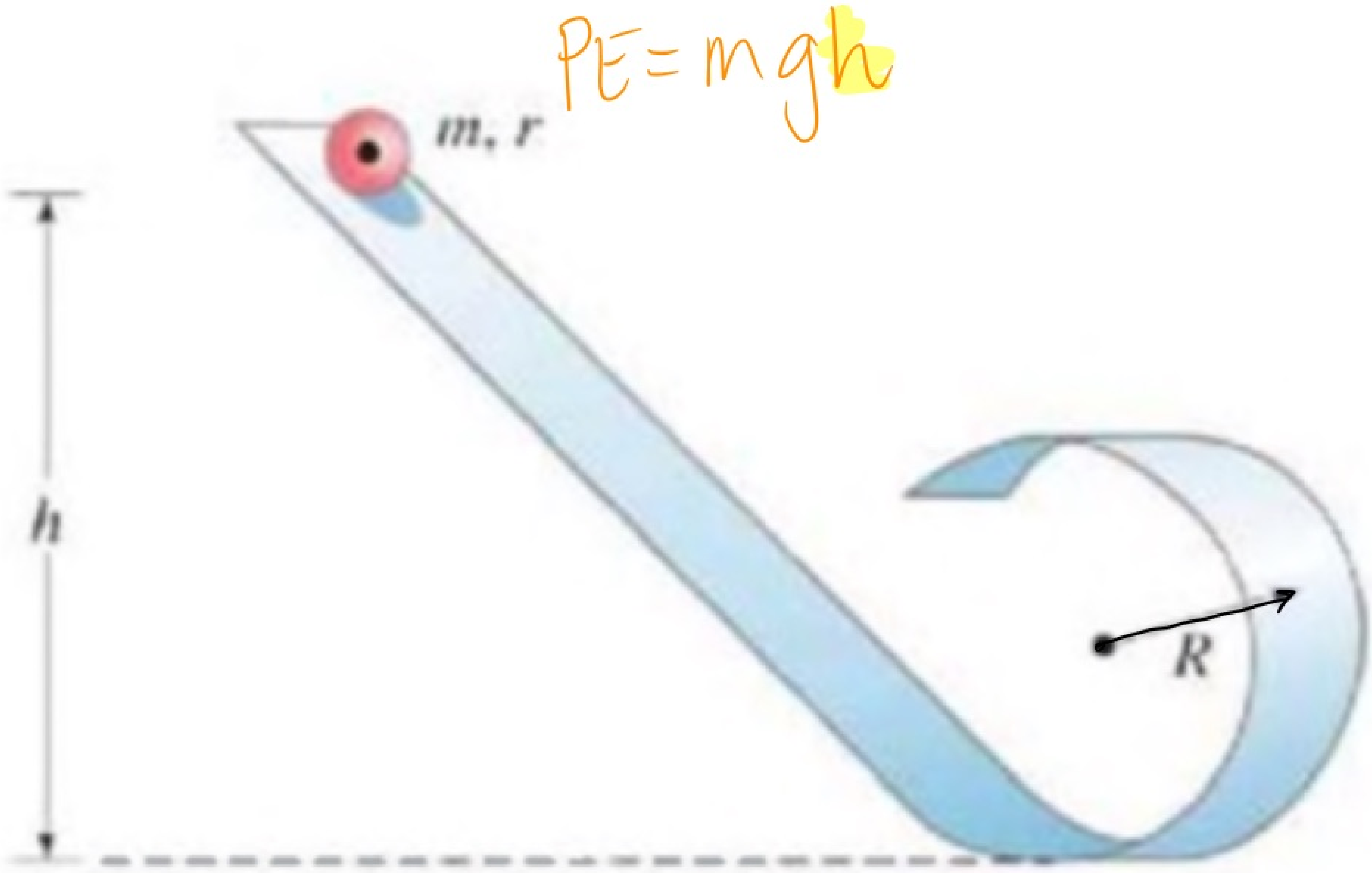
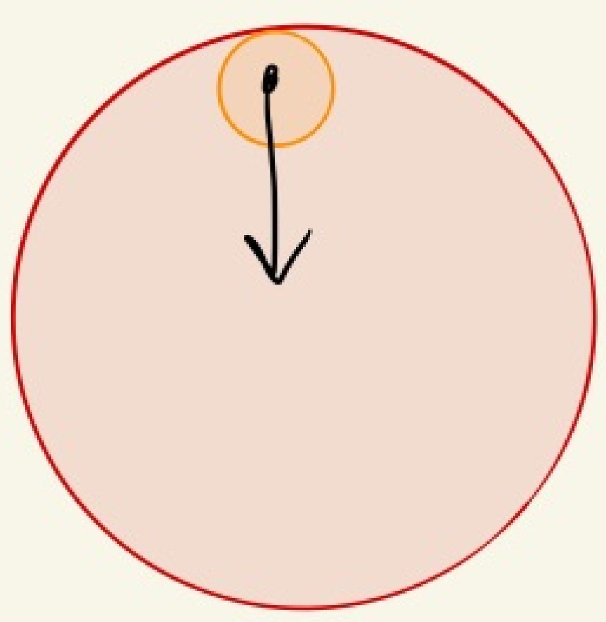


FIGURE 10–67 Problem 94.



$$\cancel{F_N}^{\,0} + mg = F_c = \frac{mv^2}{r} \leftarrow R$$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r} \leftarrow$$

$$v_{TL} = \sqrt{g(R-r)}$$

$$E_{height} = E_{top\ loop}$$

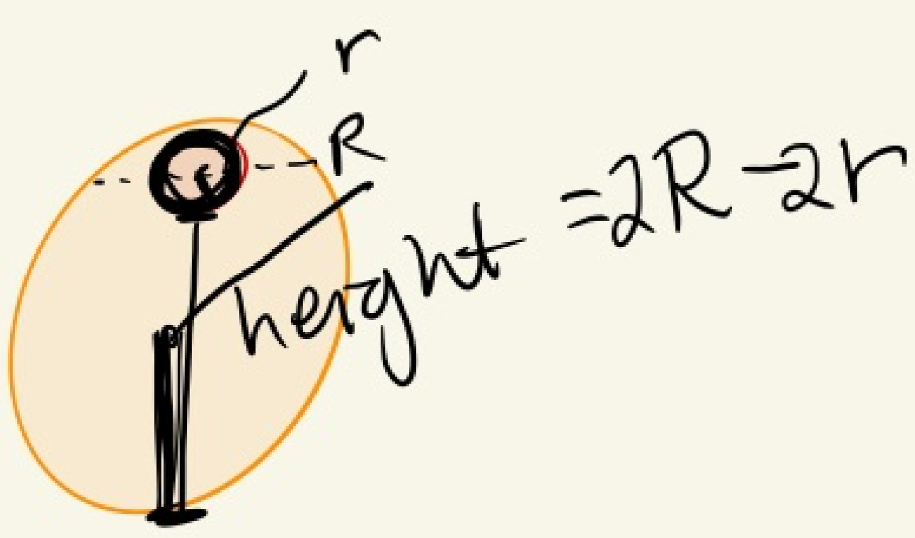
$$v = \sqrt{gr} \leftarrow$$

$$KE_{TL} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = KE_{TL} + PE_{TL}$$

$$I_{sphere} = \frac{2}{5}mr^2$$

$E_{height} = E_{top\,loop}$



$mgh = KE_{TL} + PE_{TL}$

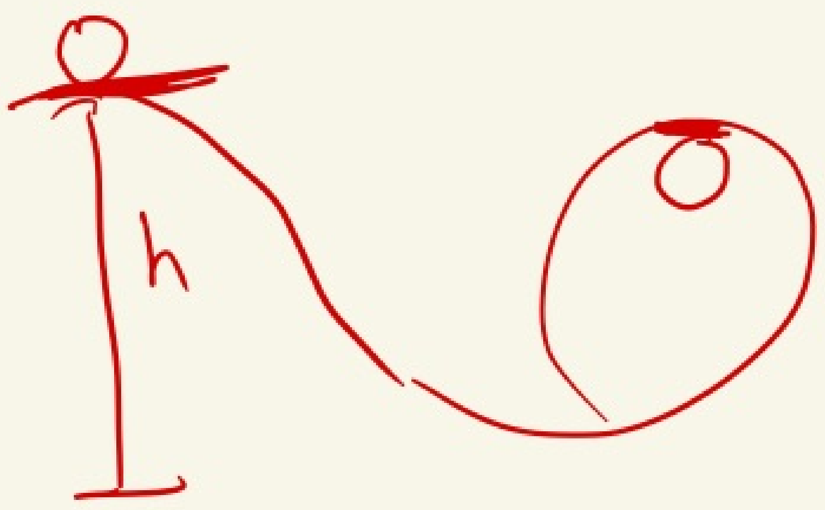
$KE_{TL} = KE_R + KE_T$

$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

$I = \frac{2}{5} mr^2$

$PE_{TL} = mgh = mg(2R-2r)$

$PE_{TL} = 2mg(R-r)$ ✓



$\cancel{m}gh = \frac{1}{2}\left(\frac{2}{5}\cancel{m}r^2\right)\omega^2 + \frac{1}{2}(\cancel{m}v^2) + 2\cancel{m}g(R-r)$

$gh = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5} \cdot r^2\omega^2 + \frac{1}{2} v_{TL}^2 + 2g(R-r)$

$\omega = \frac{v}{r}$ $\quad \left(\frac{v_{TL}}{r}\right)^2$

$gh = \frac{1}{5} \cancel{r^2} \frac{v_{TL}^2}{\cancel{r^2}} + \frac{1}{2} v_{TL}^2 + 2g(R-r)$

$v_{TL} = \sqrt{gR}$

$gh = \frac{1}{5} v_{TL}^2 + \frac{1}{2} v_{TL}^2 + 2g(R-r)$

$gh = \frac{1}{5}(gR) + \frac{1}{2}(gR) + 2g(R-r)$

solve for $\underline{h}$

$\cancel{g}h = \frac{7}{10}\cancel{g}R + 2\cancel{g}R - 2\cancel{g}r$

$h = \frac{7}{10}R + 2R - 2r = \frac{27}{10}R - 2r$

EXAMPLE 11–1 **Object rotating on a string of changing length.** A small mass m attached to the end of a string revolves in a circle on a frictionless tabletop. The other end of the string passes through a hole in the table (Fig. 11–3). Initially, the mass revolves with a speed $v_1 = 2.4\,\text{m/s}$ in a circle of radius $R_1 = 0.80\,\text{m}$. The string is then pulled slowly through the hole so that the radius is reduced to $R_2 = 0.48\,\text{m}$. What is the speed, v_2, of the mass now?

$L_i = L_f$

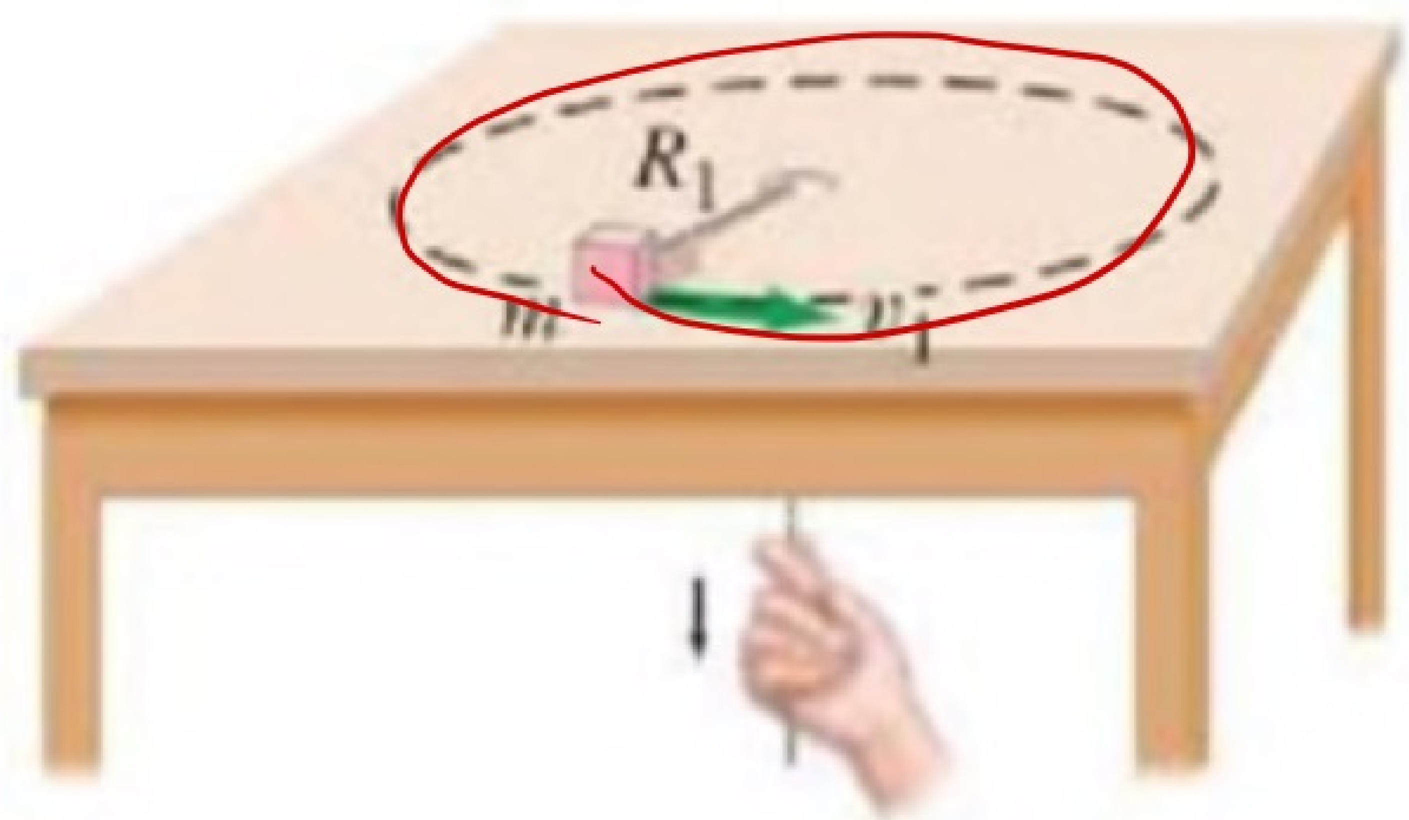
linear

$p = m \cdot v$

$L = I \cdot \omega$

rotational

FIGURE 11–3 Example 11–1.



$$I_1 \omega_1 = I_2 \omega_2$$

$$\cancel{m} R_1^2 \omega_1 = \cancel{m} R_2^2 \omega_2$$

$$R_1^2 \omega_1 = R_2^2 \omega_2$$

$$I = mR^2$$

$$v = r\omega$$

$$\omega = \frac{v}{R}$$

$$R_1^{\cancel{2}} \cdot \frac{v_1}{\cancel{R_1}} = R_2^{\cancel{2}} \frac{v_2}{\cancel{R_2}}$$

$$R_1 V_1 = R_2 V_2$$

$$V_2 = \frac{R_1 V_1}{R_2} = \frac{(0.8\,m)(2.4\,m/s)}{(0.48\,m)} = 4.0\,m/s$$

EXAMPLE 11–12 **Bullet strikes cylinder edge.** A bullet of mass m moving with velocity v strikes and becomes embedded at the edge of a cylinder of mass M and radius R_0, as shown in Fig. 11–22. The cylinder, initially at rest, begins to rotate about its symmetry axis, which remains fixed in position. Assuming no frictional torque, what is the angular velocity of the cylinder after this collision? Is kinetic energy conserved?

$\omega_{cyl.}$ after collision.

$$\boxed{L_{Bullet} = R_0 \cdot m \cdot v}$$

initial momentum of bullet

$$I_{cyel} = \frac{1}{2} M R_0^2$$

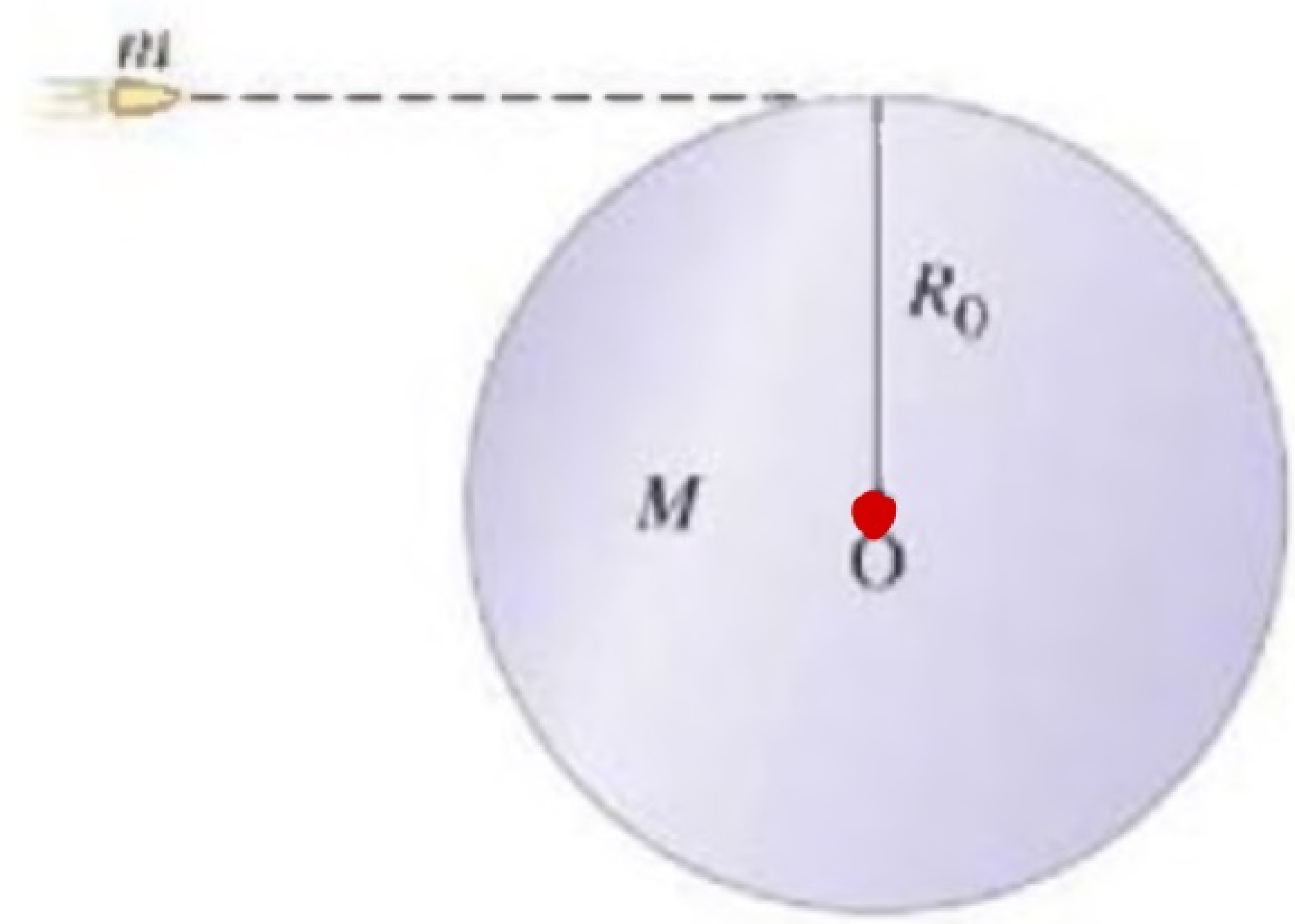


FIGURE 11–22 Bullet strikes and becomes embedded in cylinder at its edge (Example 11–12).

$$L_{after} = I\omega_{cyl.} = (I_{cyl} + I_{Bul})\,\omega$$

$$R_0 m v = I\omega_{cye+Bull} = (I_{cyc} + I_{Bul})\,\omega$$

velocity of bullet

$$\left(\frac{1}{2} M_{cy} R_0^2 + m_B R_0^2\right)\omega$$

$$R_0 m v_{Bullet} = \left(\frac{1}{2} M_{cyc} R_0^2 + M_B R_0^2\right)\omega$$

$$\omega = \frac{\cancel{R_0}\, m_{Bullet}\, v_{Bullet}}{\frac{1}{2} M_{cyc} R_0^{\cancel{2}} + M_{Bullet} R_0^{\cancel{2}}} = \frac{M_B V_B}{\frac{1}{2} M_C R_0 + M_B R_0}$$