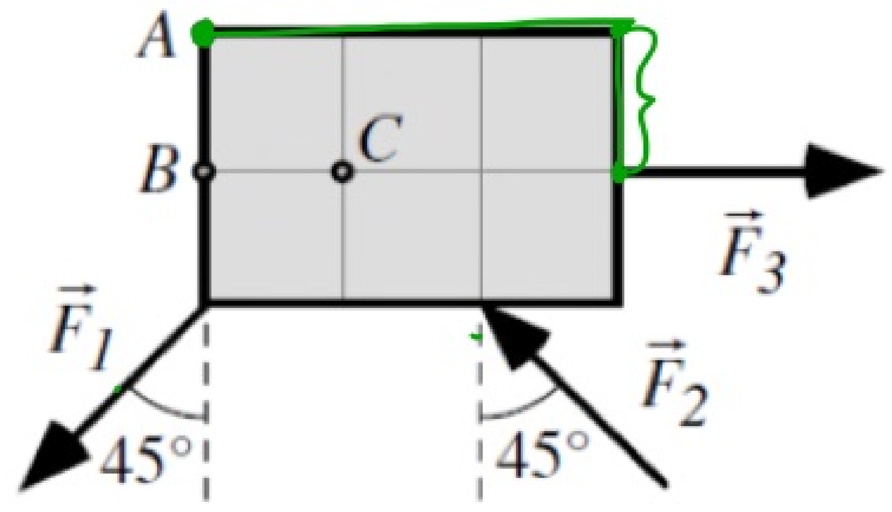
Name: KEY CVCS: AP Physics

Rotational Motion Review

Vector into page ⊗ Vector out of page ⊙ 3/2/2020

1.

Three forces of equal magnitude are applied to a 3-m by 2-m rectangle. Forces $\vec{F}_1$ and $\vec{F}_2$ act at 45° angles to the vertical as shown, while $\vec{F}_3$ acts horizontally.




**(a) Is the torque by $\vec{F}_1$ about point $A$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** i

**Explain your reasoning.**

**(b) Is the torque by $\vec{F}_1$ about point $B$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** i

**Explain your reasoning.**

**(c) Is the torque by $\vec{F}_1$ about point $C$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** Ø

**Explain your reasoning.**

**(d) Is the torque by $\vec{F}_2$ about point $A$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** Ø

**Explain your reasoning.**

**(e) Is the torque by $\vec{F}_2$ about point $B$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** CCW ii

**Explain your reasoning.**

**(f) Is the torque by $\vec{F}_2$ about point $C$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** Ø

**Explain your reasoning.**

**(g) Is the torque by $\vec{F}_3$ about point $A$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** CCW

**Explain your reasoning.**

**(h) Is the torque by $\vec{F}_3$ about point $B$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** Ø

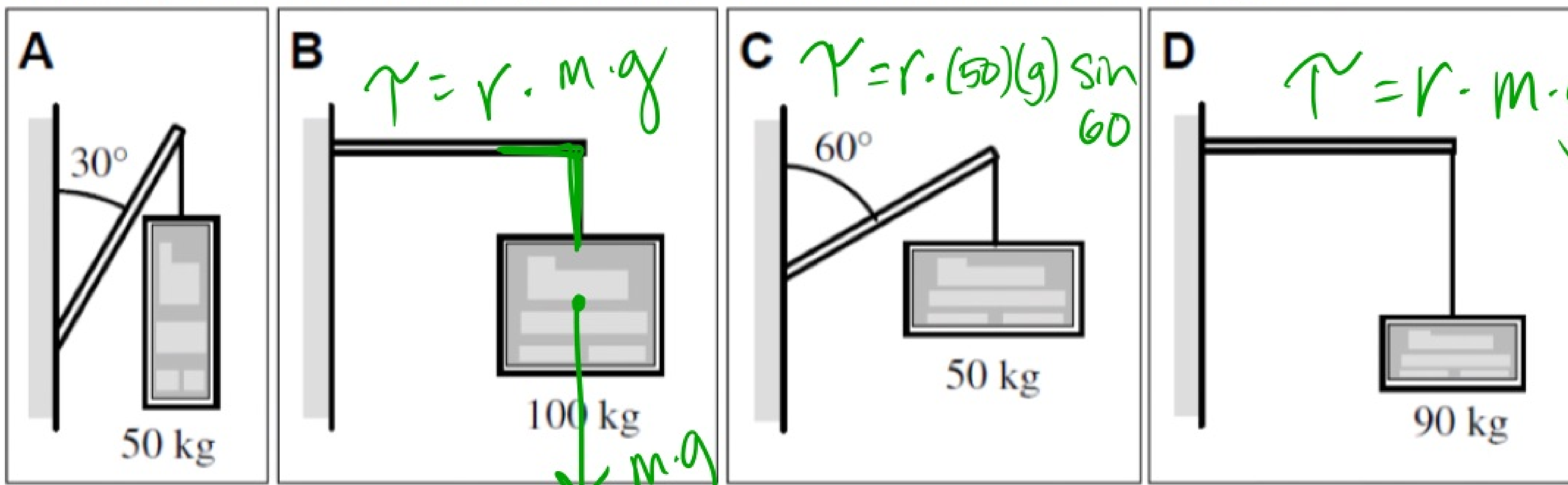
**Explain your reasoning.**

**(i) Is the torque by $\vec{F}_3$ about point $C$ (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** Ø

**Explain your reasoning.**

Signs are suspended from equal-length rods on the side of a building. For each case, the mass of the rod compared to the mass of the sign is small and can be ignored. The mass of the sign is given in each figure. In Cases B and D, the rod is horizontal; in the other cases, the angle that the rod makes with the vertical is given.

$\tau = r \cdot 50 \cdot g \cdot \sin 30$




**Rank the magnitude of the torque the signs exert about the point at which the rod is attached to the side of the building.**

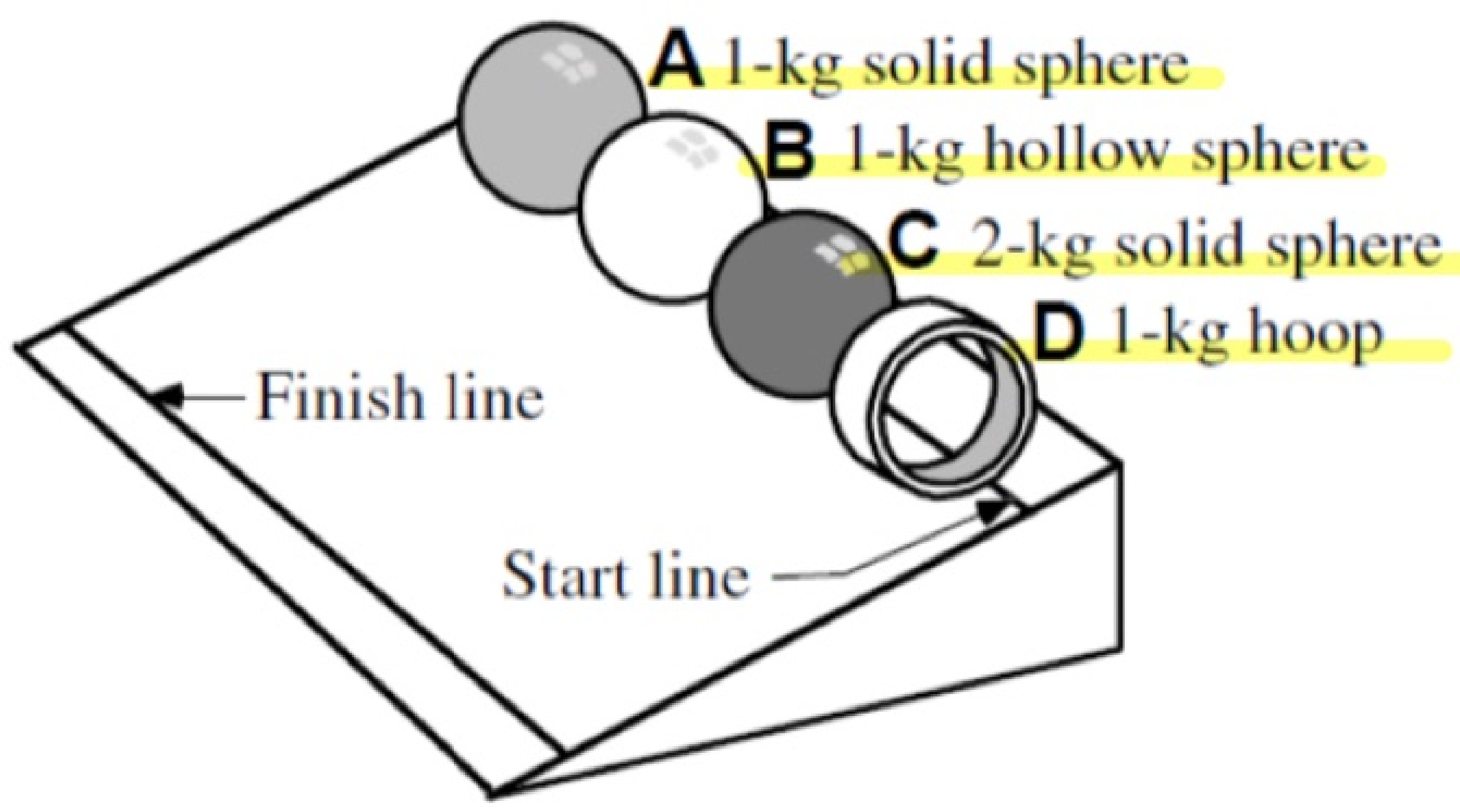
| 1 Greatest | 2 | 3 | 4 Least | OR | All the same | All zero | Cannot determine |
|---|---|---|---|---|---|---|---|
| B > | D > | C > | A | | | | |

**Explain your reasoning.**

$\tau = r \cdot F \cdot \sin\theta$ $\theta$ is between $r$ & $F$

3.

Four objects are placed in a row at the same height near the top of a ramp and are released from rest at the same time. The objects are (i) a 1-kg solid sphere; (ii) a 1-kg hollow sphere; (iii) a 2-kg solid sphere; and (iv) a 1-kg thin hoop. All four objects have the same diameter, and the hoop has a width that is one-quarter its diameter. The time it takes the objects to reach the finish line near the bottom of the ramp is recorded. The moment of inertia for an axis passing through its center of mass for a solid sphere is $\frac{2}{5}MR^2$; for a hollow sphere it is $\frac{2}{3}MR^2$; and for a hoop it is $MR^2$.




**Rank the four objects from fastest (shortest time) down the ramp to slowest.**

| 1 Fastest | 2 | 3 | 4 Slowest | OR | All the same | Cannot determine |
|---|---|---|---|---|---|---|
| A = | C | B > | D | | | |

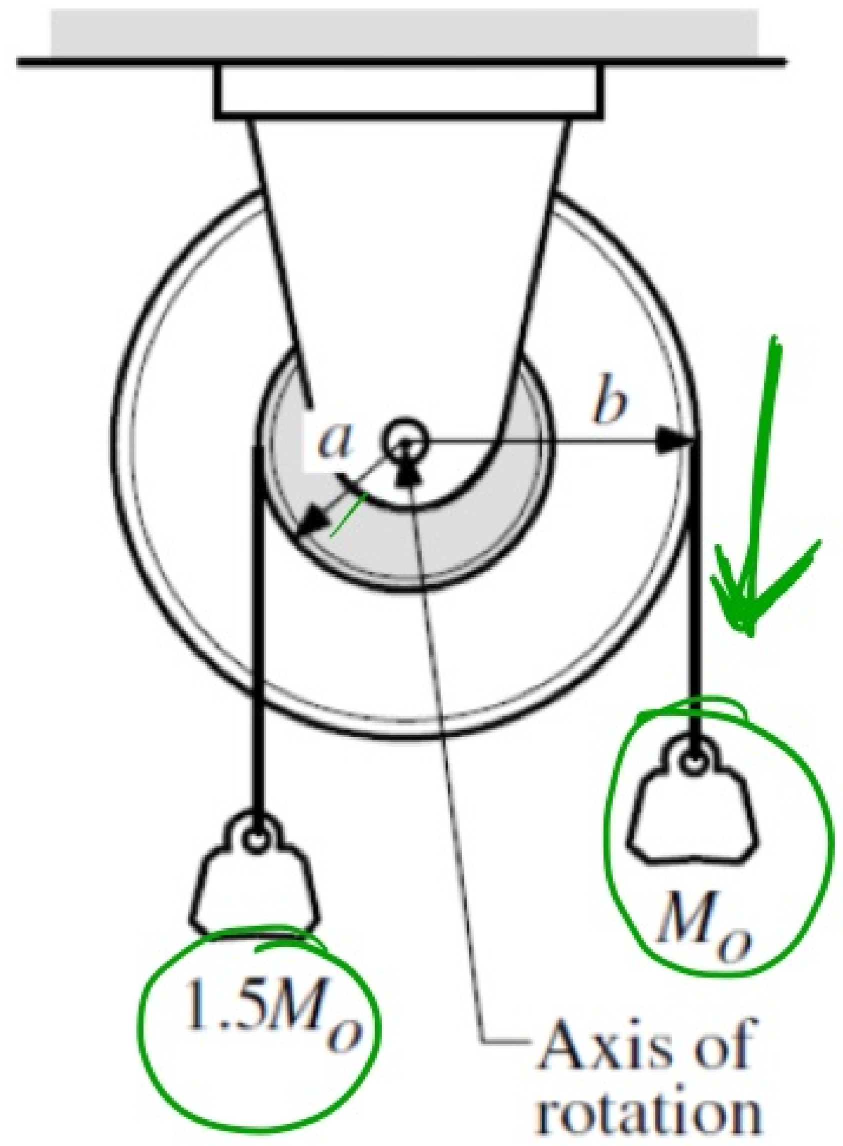
**Explain your reasoning.**

$I_A = \frac{2}{5}MR^2$ $I_B = \frac{2}{3}MR^2$ $I_C = \frac{2}{5}MR^2$

$I_D = MR^2$

Two pulleys with different radii (labeled $a$ and $b$) are attached to one another so that they rotate together. Each pulley has a string wrapped around it with a weight hanging from it. The pulleys are free to rotate about a horizontal axis through the center. The radius of the larger pulley is twice the radius of the smaller one ($b = 2a$). A student describing this arrangement states:

"The larger mass is going to create a counterclockwise torque and the smaller mass will create a clockwise torque. The torque for each will be the weight times the radius, and since the radius for the larger pulley is double the radius of the smaller, and the weight of the heavier mass is less than double the weight of the smaller one, the larger pulley is going to win. The net torque will be clockwise, and so the angular acceleration will be clockwise."



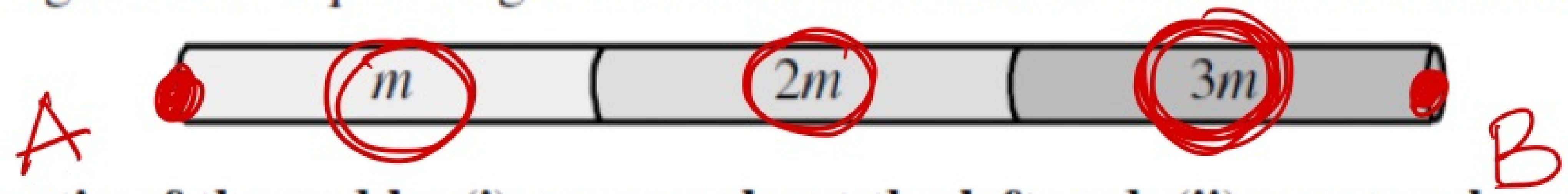

**What, if anything, is wrong with this contention? If something is wrong, explain how to correct it. If this contention is correct, explain why.**

$\tau = r \cdot F$ $\quad F = mg$

$\tau_b = M_o \cdot b = 2M_o a g$ $\quad \tau_a = a \cdot 1.5 M_o g$

5.

A rod is made of three segments of equal length with different masses. The total mass of the rod is $6m$.




**Will the moment of inertia of the rod be (i) *greater* about the left end, (ii) *greater* about the right end, or (iii) *the same* about both ends?** _____

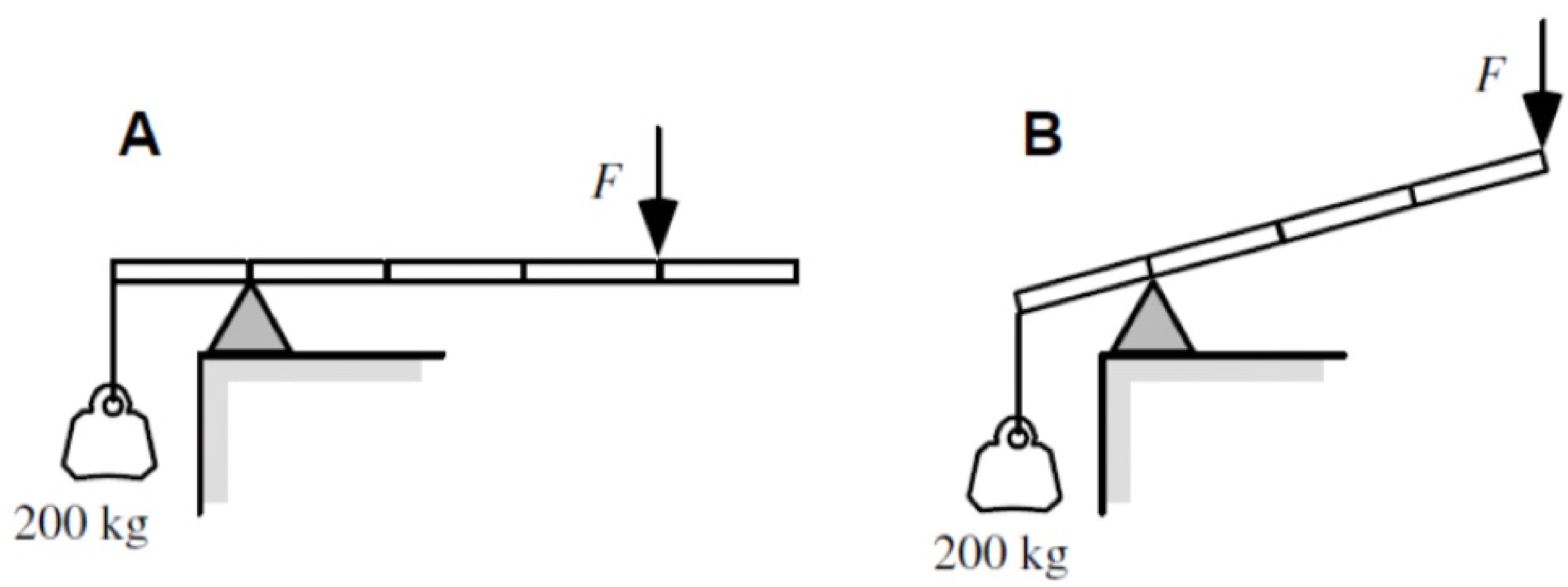
**Explain your reasoning.** i $\quad I = Mr^2$

greater about the left end

$I \uparrow$

6.

In both cases, a massless rod is supported by a fulcrum, and a 200-kg hanging mass is suspended from the left end of the rod by a cable. A downward force $F$ keeps the rod at rest. The rod in Case A is 50 cm long, and the rod in Case B is 40 cm long. (Each rod is marked at 10-cm intervals.)

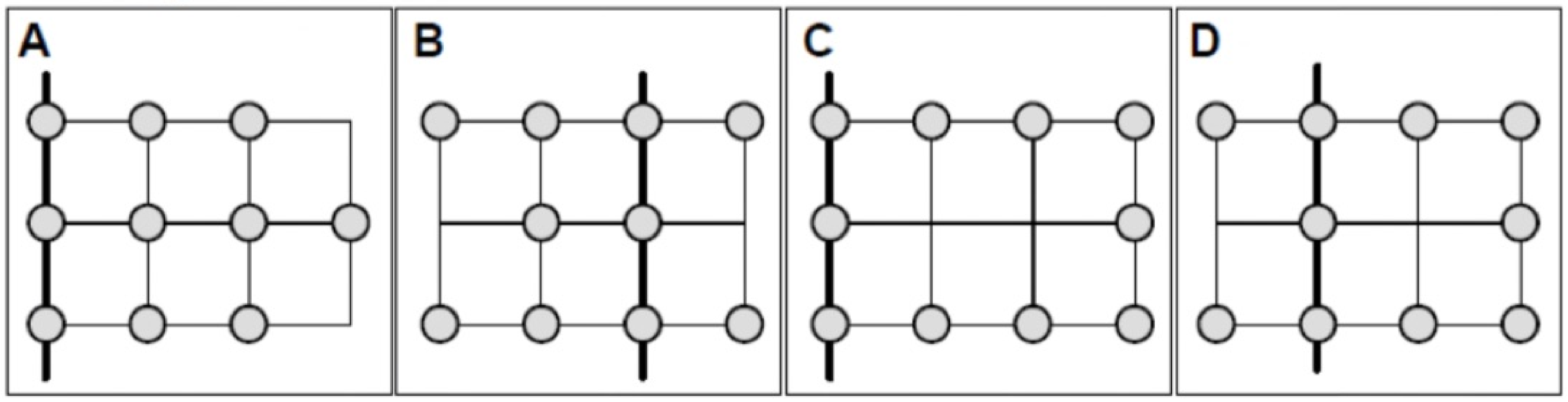



**Will the magnitude of the vertical force $F$ exerted on the rod be (i) *greater* in Case A, (ii) *greater* in Case B, or (iii) *the same* in both cases?** _____

**Explain your reasoning.** Same

7.

Each of the ten point masses in each case is identical. The solid line in each figure represents an axis about which the masses are going to be rotated. The point masses are fixed together so that they all maintain the arrangements shown while being rotated.




**Rank these arrangements on how hard it will be to start the systems rotating.**

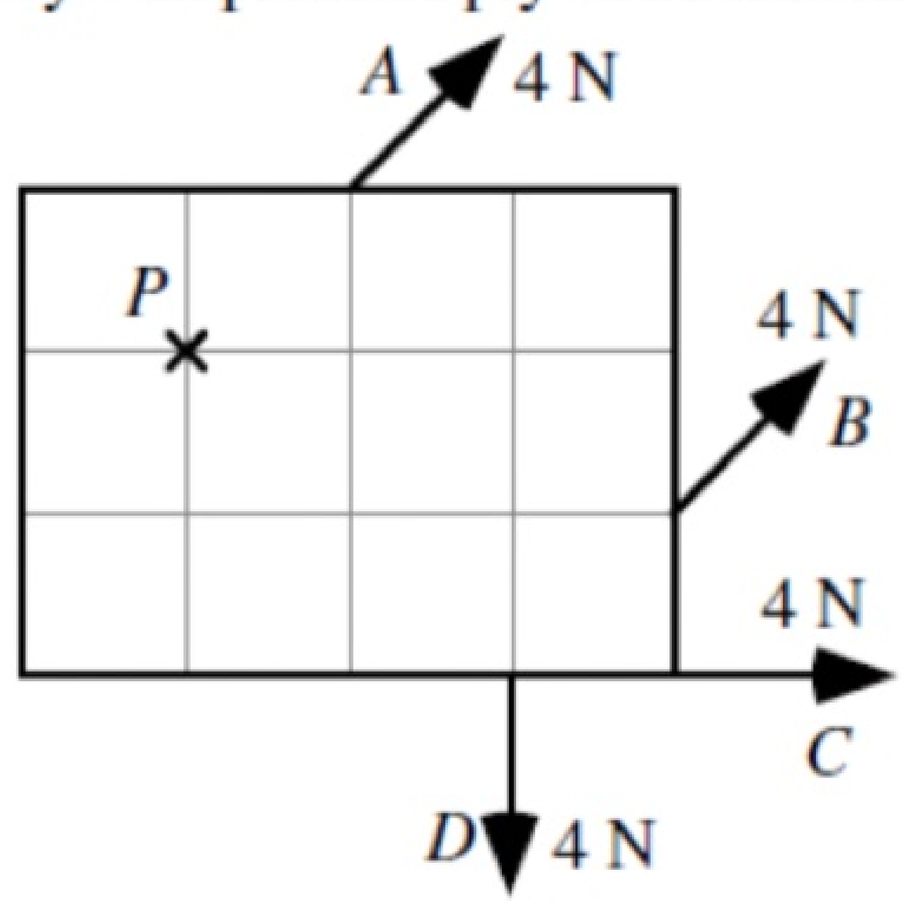
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| C > | A > | D > | B | OR | | | |
| 1 Greatest | 2 | 3 | 4 Least | | All the same | All zero | Cannot determine |

**Explain your reasoning.**

The larger amount of I the more difficult it will be to rotate.

8.

Four 4-Newton forces (*A–D*) act on a 3 m by 4 m piece of plywood that has a pivot point at *P*.

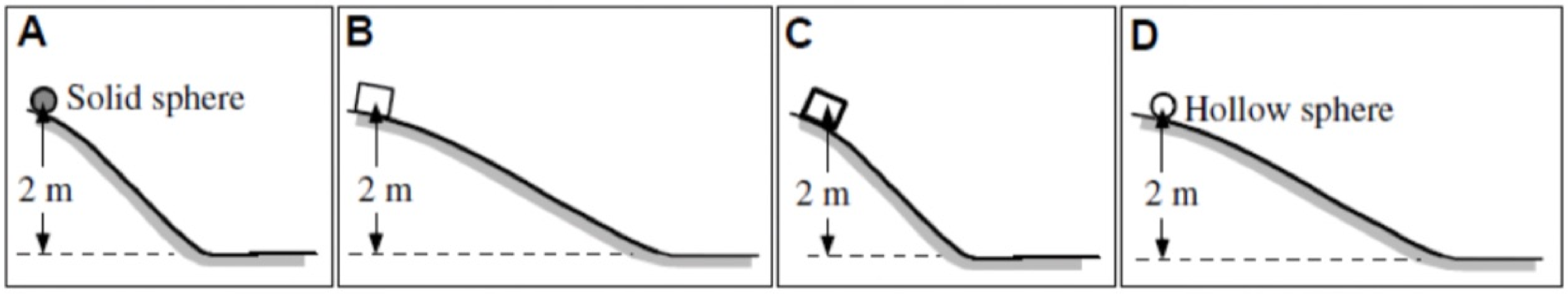



**Will the plywood rotate about the pivot point *P* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *not at all*? _____**
**Explain your reasoning.**

CCW

The net $\tau$ is the sum of the 4 individual $\tau$. Force B exerts no torque, and C & D exert equal and opposite torques. Therefore, the torque due to Force B causes the rotation.

In each case, a 1-kg object is released from rest on a ramp at a height of 2 m from the bottom. All of the spheres roll without slipping, and the blocks slide without friction. The ramps are identical in Cases A and C. The ramps in Cases B and D are identical and are not as steep as the others.




**Rank the speed of the objects when they reach the horizontal surface at the bottom of the ramp.**

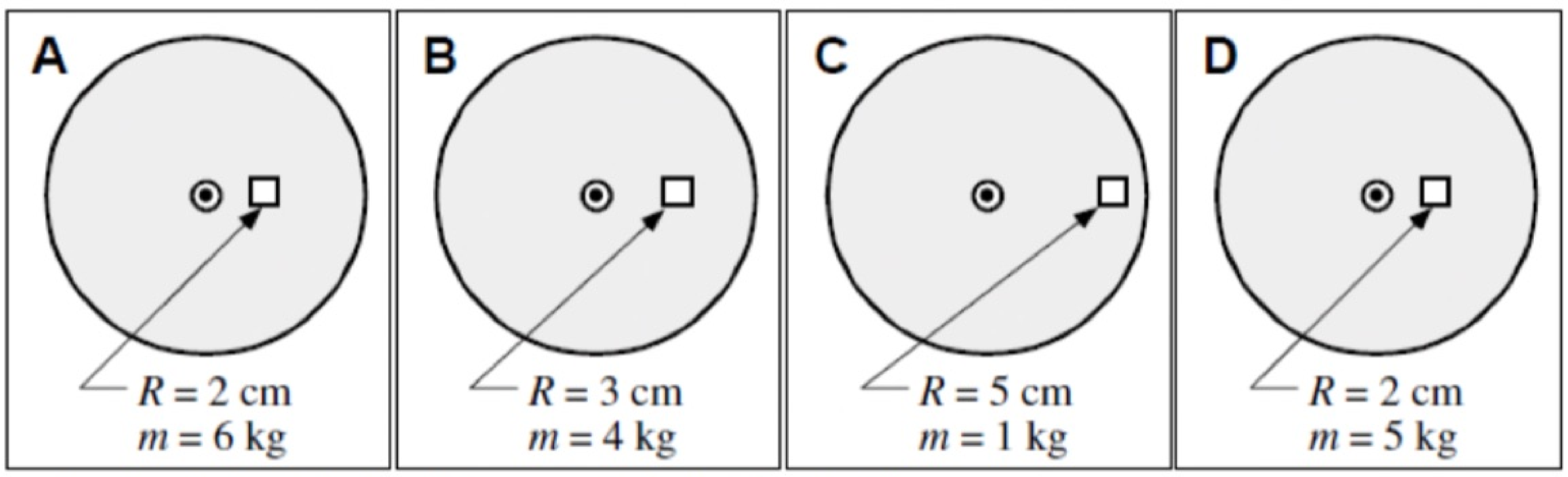
| 1 Greatest | 2 | 3 | 4 Least | OR | All the same | All zero | Cannot determine |
|---|---|---|---|---|---|---|---|
| B = | C > | A > | D | | | | |

**Explain your reasoning.**

All start w/ same Potential Energy
some is converted KErot & KEtrans

10.

A block is placed on a rotating disc and moves in a circular path. The discs have the same rotation rate in each case, but the masses of the blocks and their distance from the center varies.




**Rank the magnitude of the frictional force on blocks by the discs.**

| 1 Greatest | 2 | 3 | 4 Least | OR | All the same | All zero | Cannot determine |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

**Explain your reasoning.**

$$F_{fr} = F_C = \frac{mv^2}{r}$$

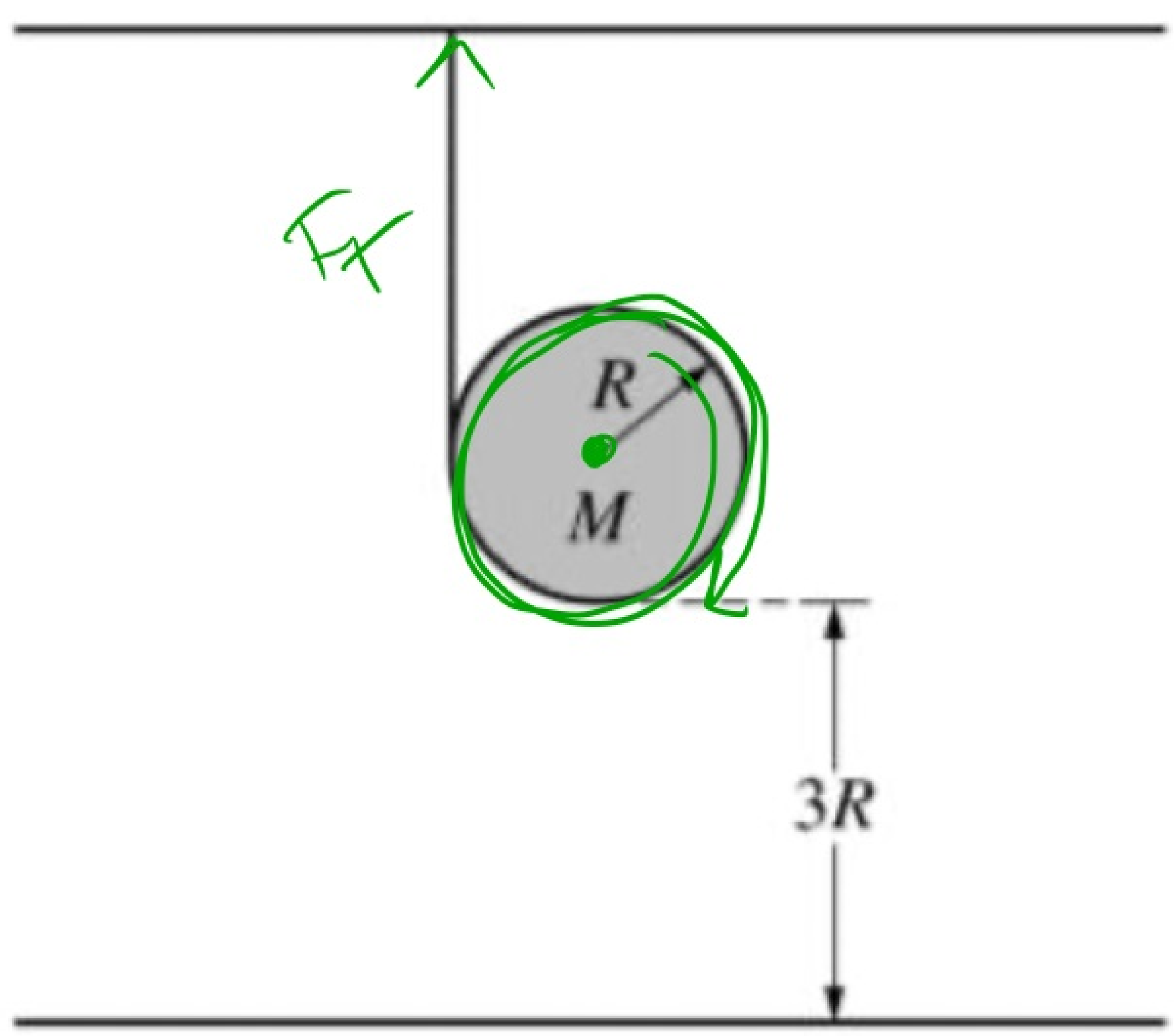
11. The angular velocity of a rotating disk with a radius of 2 m decreases from 6 rads per second to 3 rads per second in 2 seconds. What is the linear acceleration of a point on the edge of the disk during this time interval?

A) Zero
B) $-3\ m/s^2$
C) $-3/2\ m/s^2$
D) $3/2\ m/s^2$
E) $3\ m/s^2$

12. A 4 kg object moves in a circle of radius 8 m at a constant speed of 2 m/s. What is the angular momentum of the object with respect to an axis perpendicular to the circle and through its center?

A) 2 N•s
B) 6 N•m/kg
C) 12 kg•m/s
D) 24 $m^2/s$
E) 64 kg•$m^2/s$

13.



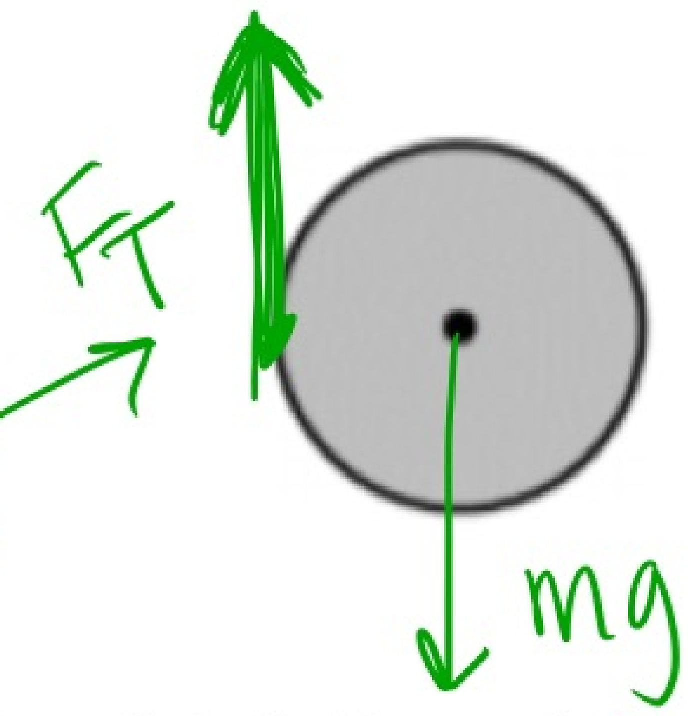

A thin uniform disk of mass M and radius R has a string wrapped around its edge and attached to the ceiling. The bottom of the disk is at a height 3R above the floor, as shown above. The disk is released from rest. The rotational inertia of a disk around its center is 2 I = MR 2 .

$I = MR^2/2$

(a) On the circle below that represents the disk, draw and label the forces (not components) that act on the disk.

Each force must be represented by a distinct arrow starting on, and pointing away from, the disk, beginning at the point where the force is exerted on the disk. The dot is at the center of the disk.

$\tau = r \cdot F_T$




$\boxed{\Sigma F = Mg - F_T = ma}$

?

(b) When released from rest, the disk falls and the string unwinds. The force the string exerts on the disk is Ft, and the gravitational force exerted on the disk is Fg . Which of the following expressions correctly relates FT and Fg as the disk falls?

__X__ Ft < Fg ____ Ft = Fg ____ Ft > Fg

Justify your answer.

Since it is accelerating downwards

(c) Express all answers in terms of M, R, and physical constants, as appropriate.

i. Derive an expression for the acceleration a of the disk as it falls.

$\tau = rF_T$ $\boxed{a = r \cdot \alpha}$ $\alpha = \frac{a}{R}$ $Mg - F_T = ma$

$\tau = I\alpha$ $\cancel{R}F_T = \frac{1}{2} \cdot MR^{\cancel{2}} \cdot \frac{a}{\cancel{R}} = \frac{1}{2}M\cancel{R}a$

$\boxed{F_T = \frac{1}{2}ma}$

$\cancel{M}g - \frac{1}{2}\cancel{M}a = \cancel{m}a$ $\frac{3}{2}a = g$ $a = \frac{2}{3}g$

ii. Derive an expression for the time (t) that it takes the disk to reach the ground.

$h = 3R$ #3 $3R = \cancel{v_i}^0 \Delta t + \frac{1}{2}a\Delta t^2$

$\omega = \alpha \cdot t$

$3R = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}g\right)t^2$ $9R = gt^2$ $t^2 = \frac{9R}{g}$ $\frac{\cancel{m}\sqrt{s^2}}{\cancel{m}/s^2} = s$

$\boxed{t = 3\sqrt{\frac{R}{g}}}$

iii. Derive an expression for the rotational kinetic energy $K_{rot}$ of the disk at the instant it reaches the ground.

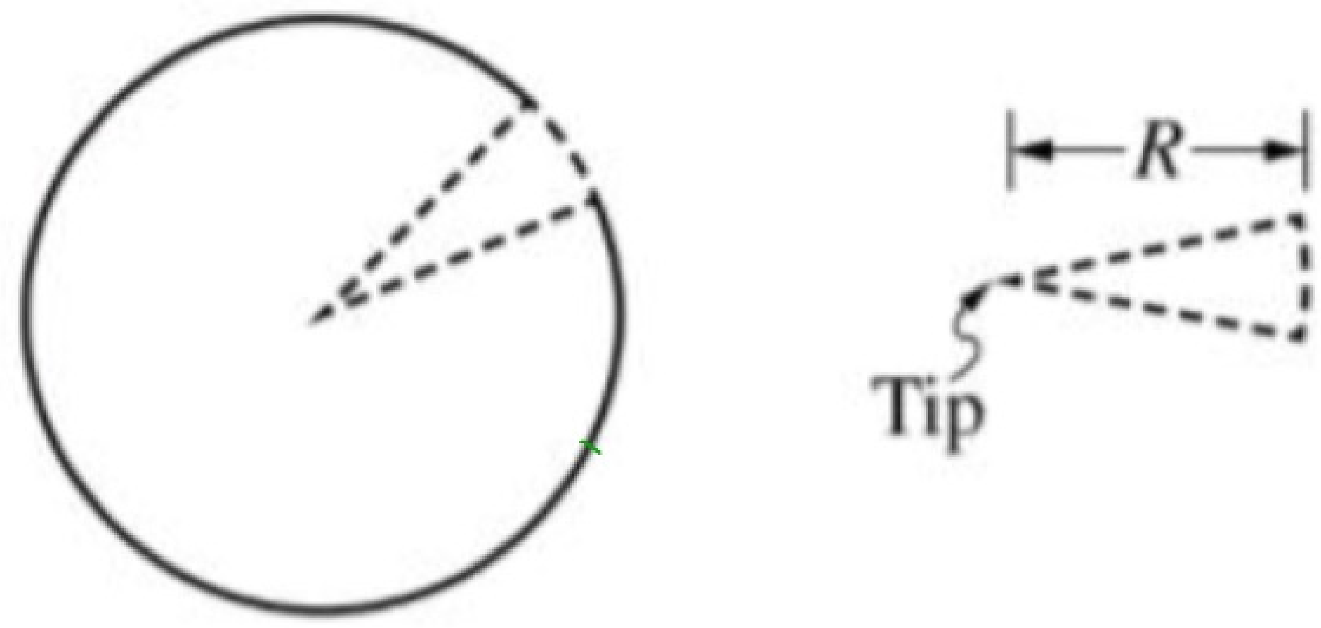
$KE_{rot} = \frac{1}{2}I\omega^2$ $PE = KE_t + KE_{Rot}$

$mgh = \frac{1}{2}mv^2 + \boxed{\frac{1}{2}I\omega^2}$ $\boxed{KE_R = MgR}$

$KE_{Rot} = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(\alpha t)^2 =$

$\left(\frac{1}{4}MR^2\right)\left(\frac{a}{R}\right)^2\left(3\sqrt{\frac{R}{g}}\right)^2 = \frac{1}{4}M\cancel{R^2} \cdot \frac{a^2}{\cancel{R^2}} 9 \cdot \frac{R}{g} = \frac{1}{4}ma^2 \cdot \frac{9R}{g}$

14. CALCULUS ONLY




Note: Figure not drawn to scale.

A very narrow wedge is cut out of the thin uniform disk of mass M, as shown above. If r is the distance from the tip of the wedge, then the linear mass density of the wedge can be expressed as follows:

$$\lambda(r) = \frac{Mr}{25R^2}.$$

i. Using integral calculus, derive an expression for the rotational inertia of the wedge around its tip.

$m = \lambda r \qquad dm = \lambda dr = \frac{Mr}{25R^2} dr$

$I = \int r^2 dm$

$I = \int r^2 \frac{Mr}{25R^2} dr = \frac{M}{25R^2} \int r^3 dr$

$I = \frac{M}{25R^2} \int_0^R r^3 dr = \frac{M}{25R^2}\left[\frac{r^4}{4}\right]_0^R = \frac{M}{25R^2}\left(\frac{R^4}{4} - 0\right) = \boxed{\frac{1}{100} MR^2}$

ii. Derive an expression for the rotational inertia of the modified disk (i.e., the disk after the narrow wedge is cut out) around its original center.

$I_{tot} = I_{disk} - I_{wedge} = \frac{1}{2} MR^2 - \frac{1}{100} MR^2 = \boxed{\frac{49}{100} MR^2}$